Measures of variability

- Measures of central tendency: typical or average value for a sample or population
- Can be misleading
- How are values distributed

Measures of variability

- How much variability is there, in a sample, or in a population?
- Crude measure:
  \[ \text{Range} = \text{highest value minus lowest value} \]
- More sophisticated measure of variability:
  \( \text{Standard deviation} \)
Standard deviation

• Average of the differences between each individual score and mean of all scores?
• How far is average score from the mean?
• How scattered are the data?

\[
S = \sqrt{\frac{\sum (Y - \bar{Y})^2}{n}}
\]

Check precedence order for calculation!
Standard deviation: grouped data

\[ S = \sqrt{\frac{\sum f(Y - \bar{Y})^2}{n}} \]

Check precedence order for calculation!

Standard deviation: example

• Find mean:

\[ \bar{Y} = \frac{\sum fY}{n} \]

= 1663/76 = 21.9
Standard deviation example:
scores on a midterm

<table>
<thead>
<tr>
<th>interval</th>
<th>midpt.</th>
<th>f</th>
<th>fY</th>
<th>$Y - \bar{Y}$</th>
<th>$(Y - \bar{Y})^2$</th>
<th>$f(Y - \bar{Y})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>3</td>
<td>x 0</td>
<td>= 0</td>
<td>-18.9</td>
<td>357.21</td>
<td>0</td>
</tr>
<tr>
<td>6-10</td>
<td>8</td>
<td>x 1</td>
<td>= 8</td>
<td>-13.9</td>
<td>193.21</td>
<td>193.21</td>
</tr>
<tr>
<td>11-15</td>
<td>13</td>
<td>x 6</td>
<td>= 78</td>
<td>-8.9</td>
<td>79.21</td>
<td>475.26</td>
</tr>
<tr>
<td>16-20</td>
<td>18</td>
<td>x 19</td>
<td>= 342</td>
<td>-3.9</td>
<td>15.21</td>
<td>288.99</td>
</tr>
<tr>
<td>21-25</td>
<td>23</td>
<td>x 33</td>
<td>= 759</td>
<td>1.1</td>
<td>1.21</td>
<td>39.93</td>
</tr>
<tr>
<td>26-30</td>
<td>28</td>
<td>x 17</td>
<td>= 476</td>
<td>6.1</td>
<td>37.21</td>
<td>632.57</td>
</tr>
<tr>
<td></td>
<td>76</td>
<td>1663</td>
<td></td>
<td></td>
<td></td>
<td>1629.96</td>
</tr>
</tbody>
</table>

Standard deviation:
grouped data

$$s = \sqrt{\frac{\sum f(Y - \bar{Y})^2}{n}}$$

Check precedence order for calculation!
Standard deviation: example

- Divide sum by n \( (1629.96/76) = 21.4 \)
- Take square root
- Answer: \( s = 4.6 \)

\[
S = \sqrt{\frac{\sum fY^2}{n} - \left(\frac{\sum fY}{n}\right)^2}
\]

- Mean of the squares
- Square of the means
Standard deviations:
calculating formula

<table>
<thead>
<tr>
<th>Y</th>
<th>Y²</th>
<th>f</th>
<th>fY²</th>
<th>fY</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td>1</td>
<td>64</td>
<td>8</td>
</tr>
<tr>
<td>13</td>
<td>169</td>
<td>6</td>
<td>1014</td>
<td>78</td>
</tr>
<tr>
<td>18</td>
<td>324</td>
<td>19</td>
<td>6156</td>
<td>342</td>
</tr>
<tr>
<td>23</td>
<td>529</td>
<td>33</td>
<td>17457</td>
<td>759</td>
</tr>
<tr>
<td>28</td>
<td>784</td>
<td>17</td>
<td>13328</td>
<td>476</td>
</tr>
</tbody>
</table>

76 38,019 1663

Standard deviation:
calculating formula

\[
s = \sqrt{(38,019/76) - (1663/76)^2}
\]

\[
s = \sqrt{500.25 - (21.88)^2}
\]

\[
s = \sqrt{21.5156}
\]

\[
s = 4.6
\]
Standard deviation

- If the population is normally distributed, you can estimate the percentage of the population within 1, 2, and 3 standard deviations of the mean:
  - 68% of cases lie +/- 1 s.d. of mean
  - 95% of cases lie +/- 2 s.d.’s of mean
  - 99.9% of cases lie +/- 3 s.d.’s of mean

\[
\bar{Y} = 21.9
\]