MRC-Based Relay Precoding for Cooperative AF Multi-Antenna Relay Networks with CSI

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***Abstract*—This paper investigates linear precoding designs for a cooperative amplify-and-forward (AF) network with a multi-antenna relay having complete channel state information (CSI). The focus is on both orthogonal AF (OAF) and non-orthogonal AF (NAF) protocols. The precoders at the relay are derived based on the maximum ratio combining (MRC) scheme, followed by an optimal power amplification factor to maximize the end-to-end achievable rate. For OAF, it is a concave optimization problem and the closed-form solution can be obtained using Karush-Kuhn-Tucker (KKT) conditions. However, the optimization problem for NAF is non-convex and getting globally optimal solution in closed-form is more challenging. Our approach is to investigate the achievable rate in different sub-domains of the channel matrix to upper-bound the original problem by a convex optimization problem. It is then shown that the optimal solution to the power amplification factor of the original optimization problem can be obtained in closed-form. The optimal MRC-based relay precoding vector is then established. Numerical results reveal that the proposed system achieves significant end-to-end rate gains over the conventional dual-hop AF multi-antenna as well as cooperative AF single-antenna systems.**

***Index Terms*—Amplify-and-forward relaying, precoding, achievable rate, channel state information, multiple antennas, maximum ratio combining, non-convex optimization.**

I. Introduction

Over the last decade, relaying communication [1]–[3] has been identified as one of the key techniques to improve the data rate and reliability of wireless networks. Different relaying protocols can be categorized as decode-and-forward (DF), compress-and-forward (CF) and amplify-and-forward (AF). Since the AF scheme requires the lowest complexity and provides the reasonable trade off between the practical implementation cost and the potential benefit, it can be considered as one of the most promising relaying protocols [3]–[7]. In AF relaying, the relay node simply amplifies the signal it received from the source and forwards the signal to the destination.

The fundamental limits of AF systems have been widely investigated on the dual-hop and multi-hop schemes without the direct source-destination link [8], [9]. Recent works have focused on the AF systems that take into account the direct link to maximize the degrees of freedom in the systems [3], [5]. In AF relaying, relay amplification strategy significantly depends on the type of relay-side channel state information (CSI). Recently, it has been shown in [10]–[13] that by having complete relay-side CSI, the performance of cooperative AF relaying systems can be significantly improved over the systems using channel distribution information (CDI) [2], [5] or using source-relay CSI at the relay(s) via channel inversion (CI) [14], [15]. However, the proposed cooperative AF schemes in [10]–[13] only considered single-antenna relays. While the benefits of having multiple antennas at the relay with complete CSI have been extensively explored for two-hop AF systems (please see [16] and references therein), the problem of effectively exploiting CSI in cooperative AF relaying with multi-antenna relays has not been addressed. For cooperative AF relaying, the advantages of multiple antennas were only studied for systems using CDI or CI [4], [5], [7], [14], [17].

Motivated by the above discussion, this paper investigates the design of relay precoders in a cooperative AF network with a multi-antenna relay having complete CSI to assist the transmission from the source to destination. Both the orthogonal AF (OAF) and the non-orthogonal (NAF) [14] transmission modes are considered. The proposed precoders at the relay are relied on the combination of the MRC technique and an optimal power amplification factor to maximize the end-to-end achievable rate. For the OAF protocol, we show that the optimization problem is concave and the closed-form solution can be obtained using Karush-Kuhn-Tucker (KKT) conditions. For the NAF protocol, the optimization problem is non-convex and obtaining the globally optimal solution is not straightforward. To overcome this drawback, we propose a new approach by examining the achievable rate in different sub-domains of the channel matrix to establish a convex optimization problem. The optimal MRC-based relay precoding vector is then obtained in closed-form. Numerical results show that the proposed system with complete relay-side CSI achieves significant rate improvement over the conventional dual-hop AF multi-antenna as well as cooperative AF single-antenna systems.

The remainder of this paper is organized as follows. Section II introduces the cooperative AF protocols of interest. The optimal MRC-based relay precoding vector is derived in Section III. Section IV presents numerical results to confirm our analysis. Finally, Section V concludes the paper.

II. System Model

The considered half-duplex AF system, shown in Fig.1, consists of a N-antenna relay (R) assisting the transmission

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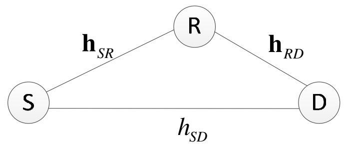

Fig. 1. The cooperative AF system with the direct source-destination link.

from a single-antenna source (S) to a single-antenna destination (D). The instantaneous complex channel gains of the S-R, R-D and S-D links can be denoted, respectively, as $\mathbf{h}_{SR} = [h_{SR_1}, ...h_{SR_N}]^T$, $\mathbf{h}_{RD} = [h_{RD_1}, ...h_{RD_N}]$ and h_{SD}. The transmission from S to D is carried out in a sequence of cooperative frames, each consisting of two phases: the broadcasting phase and the cooperative phase. In the broadcasting phase of a given cooperative frame, S sends the signal x_1 to both R and D. The received signals at D and R in this phase are given, respectively, as

$$y_1 = \sqrt{E_S} h_{SD} x_1 + n_{d_1}, \tag{1}$$

$$\mathbf{r} = \sqrt{E_S} \mathbf{h}_{SR} x_1 + \mathbf{n}_r, \tag{2}$$

where $\sqrt{E_S}$ is a strictly positive constant related to the transmit power, $\mathbf{n}_r = [n_{r_1}, ...n_{r_N}]^T$ with $n_{r_1}, ...n_{r_N}$ and w_1 are the i.i.d zero-mean circularly symmetric Gaussian noises with variance N_0, denoted as $\mathcal{CN}(0, N_0)$.

Let $\mathbf{w} = [w_1, ...w_N]^T$ be the $N \times 1$ pre-coding vector at the relay. The received signal vector at the relay will be multiplied by the pre-coding vector before being forwarded to the destination in the cooperative phase. In the cooperative phase, the source can either transmit a new signal or remain silent, depending on whether the system operate on the OAF or NAF mode as we describe in the following.

A. OAF Mode

In the OAF mode, the source keeps silent in the cooperative phase while the relay forwards the precoded signal on all of its antennas to the destination. The received signal at the destination in this phase is given as

$$y_2 = \left(\sum_{n=1}^{N} h_{RD_n} \right) \mathbf{w}^T \mathbf{r} + n_{d_2}. \tag{3}$$

where $n_{d_2} \sim \mathcal{CN}(0, N_0)$. The input-output relationship of the OAF system can be given as

$$\mathbf{y} = \sqrt{E_S} \mathbf{H}_1 x_1 + \mathbf{n}, \tag{4}$$

where the channel matrix

$$\mathbf{H}_1 = \left[h_{SD}, \Sigma \mathbf{w}^T \mathbf{h}_{SR} \sum_{n=1}^{N} h_{RD_n} \right]^T,$$

with

$$\Sigma = \frac{1}{\sqrt{1 + \|\mathbf{w}\|^2 \|\mathbf{h}_{RD}\|^2}} \tag{5}$$

is the scalar scaling factor and $\mathbf{n} \sim \mathcal{CN}(0, N_0)$ is the equivalent noise component.

B. NAF Mode

In the NAF mode, the direct source-destination link is exploited in both transmission phases. In particular, the source transmits new signal x_2 in the cooperative phase while the relay forwards the precoded signal to the destination. The received signal at D in the second phase is given as

$$y_2 = \sqrt{E_S} h_{SD} x_2 + \left(\sum_{n=1}^{N} h_{RD_n} \right) \mathbf{w}^T \mathbf{r} + n_{d_2}. \tag{6}$$

The NAF system model can be written in matrix form as

$$\mathbf{y} = \sqrt{E_S} \mathbf{H}_2 \mathbf{x} + \mathbf{n}, \tag{7}$$

where the input vector $\mathbf{x} = [x_1, x_2]^T$, the 2×2 channel matrix $\mathbf{H}_2$ is given as

$$\mathbf{H}_2 = \begin{bmatrix} h_{SD} & 0 \\ \Sigma \mathbf{w}^T \mathbf{h}_{SR} \sum\limits_{n=1}^{N} h_{RD_n} & \Sigma h_{SD} \end{bmatrix}, \tag{8}$$

with scaling factor Σ is given in (5) and $\mathbf{n} \sim \mathcal{CN}(0, N_0)$ is the equivalent noise component.

C. MRC-based Relay Pre-coder

In this paper, similar to the previous work in [11]–[13], we assume that complete CSI of all the links are available at the destination and the relay, but not the source. Given this CSI knowledge, the approach in this paper is to employ an MRC-based precoder at the relay to exploit the benefits of CSI. In such a scheme, the pre-coding vector at the relay is formulated as $\mathbf{w} = b\bar{\mathbf{h}}_{SR}$, where b is the instantaneous power amplification factor at the relay and $\bar{\mathbf{h}}_{SR}$ is the complex conjugate of $\mathbf{h}_{SR}$.

Now, let $q_1 = \mathbb{E}[|x_1|^2]$ and $q_2 = \mathbb{E}[|x_2|^2]$. This can be interpreted as the source uses the average power of $E_S q_1$ in the broadcasting phase of both OAF and NAF modes, and it transmits the power of $E_S q_2$ in the cooperative phase of the NAF mode. Furthermore, let $E_S q_r$ be the instantaneous transmit power on each antenna at the relay, i.e.,

$$\rho q_1 \left(\mathbf{w}^T \mathbf{h}_{SR} \right)^2 + \|\mathbf{w}\|^2 = \rho q_r, \tag{9}$$

where $\rho = E_S / N_0$ is the normalized SNR. Note that q_r depends on the instantaneous channel matrix $\mathbf{H}$, where $\mathbf{H} = \mathbf{H}_1$ for OAF or $\mathbf{H} = \mathbf{H}_2$ for NAF. As such, one can write $q_r = q_r(\mathbf{H})$.

The amplification factor at the relay can then be calculated as

$$b = \sqrt{\frac{\rho q_r(\mathbf{H})}{\|\mathbf{h}_{SR}\|^2 \left(\rho q_1 \|\mathbf{h}_{SR}\|^2 + 1 \right)}}. \tag{10}$$

It is also assumed that the total long-term average transmit power on all antennas at the relay is limited by P_R, i.e.,

$$\mathbb{E}_{\mathbf{H}}[q_r(\mathbf{H})] \leq P_R. \tag{11}$$

Note that in this paper, we assume that each antenna applies the same instantaneous amplification coefficient b. Further improvement can be achieved if the instantaneous amplification coefficient is optimized for each antenna. However, as shall be shown shortly, the use of the same coefficient b, while being sub-optimal in general, can significantly boost up the rate performance.

III. Optimal MRC-based Relay Pre-coding Vector

This section focuses on the design of an optimal MRC-based precoding vector $\mathbf{w}$. The OAF protocol is first considered, followed by the NAF protocol.

A. OAF Mode

Suppose the input signal uses Gaussian codebook and given the CSI information at the destination, the conditional achievable rate between the input and the output of the OAF system can be written as [5]

$$I\left(x_1, \mathbf{y}|\mathbf{H}_1\right) = \frac{1}{2}\log\det\left(1+\rho\mathbf{H}_1^{\dagger}\mathbf{H}_1 q_1\right), \quad (12)$$

where $\dagger$ denotes the Hermitian operator and $log(.)$ is the base-2 logarithm. The optimal power amplification factor that maximizes the long-term average achievable rate of the OAF system can be obtained by solving the following optimization problem

$$\max_{q_r(\mathbf{H}_1)\geq 0} \mathbb{E}_{\mathbf{H}_1}\left[I\left(x_1, \mathbf{y}|\mathbf{H}_1\right)\right] \; s.t. \, \mathbb{E}_{\mathbf{H}_1}\left[q_r\left(\mathbf{H}_1\right)\right] \leq P_R. \quad (13)$$

For the sake of convenience, let $\alpha = \left\|\mathbf{h}_{SR}\right\|^2$, $\beta = \left\|\mathbf{h}_{RD}\right\|^2$, $\gamma = \left|h_{SD}\right|^2$, $\delta = \left[\sum\limits_{n=1}^{N}\mathrm{Re}\left(h_{RD_n}\right)\right]^2 + \left[\sum\limits_{n=1}^{N}\mathrm{Im}\left(h_{RD_n}\right)\right]^2$ and $I\left(x_1, \mathbf{y}|\mathbf{H}_1\right) = I_1\left(q_r\left(\mathbf{H}_1\right)\right)$. From (10) and (12), it follows that

$$I_1\left(q_r\left(\mathbf{H}_1\right)\right) = \frac{1}{2}log\left(1+\rho q_1\gamma + \frac{\rho^2 q_1 q_r \delta\alpha}{1+\rho q_r\beta + \rho q_1\alpha}\right). \quad (14)$$

The first and second derivatives of the objective function in (13) are calculated as

$$\frac{\partial^2\mathbb{E}_{\mathbf{H}_1}\left[I_1\left(q_r\left(\mathbf{H}_1\right)\right)\right]}{\partial q_r^2\left(\mathbf{H}_1\right)} = \frac{-a_1\left(2c_1 q_r + d_1\right)}{2\ln\left(2\right)\left[c_1 q_r^2\left(\mathbf{H}_1\right) + d_1 q_r\left(\mathbf{H}_1\right) + e_1\right]^2}, \quad (15)$$

$$\frac{\partial^2\mathbb{E}_{\mathbf{H}_1}\left[I_1\left(q_r\left(\mathbf{H}_1\right)\right)\right]}{\partial q_r^2\left(\mathbf{H}_1\right)} = \frac{-a_1\left(2c_1 q_r + d_1\right)}{2\ln\left(2\right)\left[c_1 q_r^2\left(\mathbf{H}_1\right) + d_1 q_r\left(\mathbf{H}_1\right) + e_1\right]^2}, \quad (16)$$

where

$$\begin{aligned} a_1 &= \rho^2 q_1\alpha\delta\left(\rho q_1\alpha + 1\right), \\ c_1 &= \rho^2\beta\left(\rho\gamma q_1 x + \rho\delta q_1\alpha + \beta\right) \\ d_1 &= \rho^2 q_1\left[2\beta\left(\rho q_1\gamma\alpha + \gamma + \alpha\right) + \delta\alpha\left(1+\rho q_1 z\right)\right] + 2\rho\beta, \\ e_1 &= \left(1+\rho q_1\gamma\right)\left(1+\rho q_1\alpha\right)^2. \end{aligned} \quad (17)$$

It is then straightforward to verify that the Hessian matrix of the objective function in (13) is diagonal with strictly negative diagonal entries given in (16). Given that the constraint in (13) is linear, this is a *concave* optimization problem that can be solved using Karush-Kuhn-Tucker (KKT) conditions to yield the closed-form optimal solution $q_r^*\left(\mathbf{H}_1\right)$. In particular, the Lagrangian of (13) can be given as

$$\begin{aligned} \mathcal{L}\left(q_r\left(\mathbf{H}_1\right), \lambda_1\right) = &-\mathbb{E}_{\mathbf{H}_1}\left[I_1\left(q_r\left(\mathbf{H}_1\right)\right)\right] \\ &-\lambda_1\left(P_R - \mathbb{E}_{\mathbf{H}_1}\left[q_r\left(\mathbf{H}_1\right)\right]\right), \end{aligned} \quad (18)$$

where $\lambda_1 \geq 0$ is the Lagrangian multiplier. Differentiating the Lagrangian in (18) we obtain

$$\frac{\partial\mathcal{L}\left(q_r\left(\mathbf{H}_1\right), \lambda_1\right)}{\partial q_r\left(\mathbf{H}_1\right)} = \lambda_1 - \frac{\partial I_1\left(q_r\left(\mathbf{H}_1\right)\right)}{\partial q_r\left(\mathbf{H}_1\right)}, \quad (19)$$

with the derivative given in (15). By equating the gradient of the above Lagrangian to zero and solving for $q_r\left(\mathbf{H}_1\right)$, the globally optimal solution $q_r^*\left(\mathbf{H}_1\right)$ for the OAF mode is obtained as

$$q_r^*\left(\mathbf{H}_1\right) = \left(\frac{-d_1 + \sqrt{d_1^2 - 4c_1\left(e_1 - \frac{a_1}{2\ln(2)\lambda_1^*}\right)}}{2c_1}\right)^+, \quad (20)$$

where $(x)^+ = max(0, x)$ and $\lambda_1^* > 0$ is a unique constant satisfying $\mathbb{E}_{\mathbf{H}_1}[q_r^*\left(\mathbf{H}_1\right)] = P_R$ and can be found by bisection method. The MRC-based pre-coding vector $\mathbf{w}$ at the relay for the OAF mode, which is denoted as $\mathbf{w}_{OAF}^*$ is finally expressed as

$$\mathbf{w}_{OAF}^* = \left(\sqrt{\frac{\rho q_r^*\left(\mathbf{H}_1\right)}{\rho q_1\alpha^2 + \alpha}}\right)\bar{\mathbf{h}}_{SR}. \quad (21)$$

B. NAF Mode

When one has NAF relaying, the conditional achievable rate between the input and output of the NAF system, given the complete CSI at the destination, is calculated as [5]

$$I\left(\mathbf{x}, \mathbf{y}|\mathbf{H}_2\right) = I_2\left(q_r\left(\mathbf{H}_2\right)\right) = \frac{1}{2}\log\det\left(\mathbf{I}_2 + \rho\mathbf{H}_2^{\dagger}\mathbf{H}_2\mathbf{Q}\right). \quad (22)$$

where $\mathbf{Q} = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix}$ is the input covariance matrix at the source.

The optimal power amplification factor that maximizes the long-term average achievable rate of the NAF system can be obtained by solving the following optimization problem

$$\max_{q_r(\mathbf{H}_2)\geq 0} \mathbb{E}_{\mathbf{H}_2}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right] \; s.t. \, \mathbb{E}_{\mathbf{H}_2}\left[q_r(\mathbf{H}_2)\right] \leq P_R. \quad (23)$$

From (10) and (22) we have

$$\begin{aligned} &I_2\left(q_r\left(\mathbf{H}_2\right)\right) = \\ &\tfrac{1}{2}\log\left[1+\rho q_1\gamma + \tfrac{\rho^2 q_1 q_r\alpha\delta + \rho q_2\gamma(1+\rho q_1\alpha)(1+\rho q_1\gamma)}{1+\rho q_1\alpha + \rho q_r\beta}\right]. \end{aligned} \quad (24)$$

Calculating the first and second order derivatives of the objective function in (23) we obtain

$$\frac{\partial\mathbb{E}_{\mathbf{H}_2}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right]}{\partial q_r(\mathbf{H}_2)} = \frac{a_2}{2\ln\left(2\right)\left(c_2 q_r^2 + d_2 q_r + e_2\right)}, \quad (25)$$

$$\frac{\partial^2\mathbb{E}_{\mathbf{H}_2}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right]}{\partial q_r^2(\mathbf{H}_2)} = \frac{-a_2\left(2c_2 q_r + d_2\right)}{2\ln\left(2\right)\left(c_2 q_r^2 + d_2 q_r + e_2\right)^2}, \quad (26)$$

where

$$\begin{aligned} a_2 &= \rho^3\left[\alpha q_1^2\left(\alpha\delta - \rho\beta\gamma^2 q_2\right) - \beta\gamma q_1 q_2\left(\alpha+\gamma\right)\right] \\ &+\rho^2\left(\alpha\delta q_1 - \beta\gamma q_2\right), \\ c_2 &= \rho^2\left[\beta^2 + \rho\beta q_1\left(\alpha\delta + \beta\gamma\right)\right], \\ d_2 &= \rho^2[\rho q_1 q_2\beta\gamma(\alpha+\gamma) + \rho\alpha q_1^2(\alpha\delta + 2\beta\gamma + \rho\beta\gamma^2 q_2) + \\ &q_1(\alpha\delta + 2\alpha\beta + 2\beta\gamma) + \beta\gamma q_2] + 2\beta\rho, \\ e_2 &= \left(1+\rho q_1\gamma\right)\left(1+\rho q_2\gamma\right)\left(1+\rho q_1\alpha\right)^2. \end{aligned} \quad (27)$$

It can be seen that the parameter a_2 in (27) can be positive, negative or zero depending on the SNR, power allocation at the source and the channel gains. Thus, the optimization problem in (23) is not concave and hence, the KKT conditions cannot be applied directly. To overcome this drawback, we shall derive an achievable upper-bound on the objective function in (23) and show that the modified optimization problem is indeed concave and the closed-form optimal solutions can then be obtained. Observe from (25) that $I_2\left(q_r\left(\mathbf{H}_2\right)\right)$ is a strictly increasing function of $q_r\left(\mathbf{H}_2\right)$ when $a_2 > 0$ and it is a strictly decreasing function of $q_r\left(\mathbf{H}_2\right)$ when $a_2 < 0$. Lets define the feasible domain of the problem in (23) as

$$\mathcal{F} = \left\{q_r(\mathbf{H}_2)|q_r(\mathbf{H}_2) \geq 0,\, \mathbb{E}_{\mathbf{H}_2}\left[q_r(\mathbf{H}_2)\right] \leq P_R\right\}. \quad (28)$$

Furthermore, let divide the domain of the channel matrix $\mathbf{H}_2$ into two disjoint regions as $\mathbf{H}_N = \{\mathbf{H}_2|a_2 \leq 0\}$ and $\mathbf{H}_P = \{\mathbf{H}_2|a_2 > 0\}$. It is straightforward to verify that $I_2\left(q_r\left(\mathbf{H}_2\right)\right)$ is a strictly increasing function in $\mathbf{H}_P$ and is a strictly decreasing function in $\mathbf{H}_N$, with respect to $q_r\left(\mathbf{H}_2\right)$. Let $p = \Pr\left(\mathbf{H}_2 \in \mathbf{H}_P\right)$, and since $q_r\left(\mathbf{H}_2\right) \geq 0$ for any $q_r\left(\mathbf{H}_2\right) \in \mathcal{F}$, we have

$$\begin{aligned} &\mathbb{E}_{\mathbf{H}_2}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right] = \\ &p\mathbb{E}_{\mathbf{H}_2\in\mathbf{H}_P}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right] + (1-p)\mathbb{E}_{\mathbf{H}_2\in\mathbf{H}_N}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right] \\ &\leq p\mathbb{E}_{\mathbf{H}_2\in\mathbf{H}_P}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right] + (1-p)\mathbb{E}_{\mathbf{H}_2\in\mathbf{H}_N}\left[I_2\left(0\right)\right]. \end{aligned} \quad (29)$$

The equality in (29) is achieved when $q_r\left(\mathbf{H}_2\right) = 0 \;\forall\; \mathbf{H}_2 \in \mathbf{H}_N$. Since p and $I_2\left(0\right)$ do not depend on $q_r\left(\mathbf{H}_2\right)$, the optimal solution $q_r^*\left(\mathbf{H}_2\right)$ that maximizes the average achievable rate can be obtained by solving the following optimization problem

$$\begin{aligned} &\max_{q_r(\mathbf{H}_2)\geq 0} \; \mathbb{E}_{\mathbf{H}_2\in\mathbf{H}_P}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right] \\ &\quad s.t. \;\; \mathbb{E}_{\mathbf{H}_2\in\mathbf{H}_P}\left[q_r\left(\mathbf{H}_2\right)\right] \leq P_R. \end{aligned} \quad (30)$$

Note that the problem in (30) is now concave since the Hessian matrix of the objective function is diagonal with strictly negative elements in (26) for $a_2 > 0$. Given that the constraint in (30) is linear, this problem can now be solved using the KKT conditions with the Lagrangian written as

$$\begin{aligned} \mathcal{L}\left(q_r\left(\mathbf{H}_2\right), \lambda_2\right) = &-\mathbb{E}_{\mathbf{H}_2\in\mathbf{H}_P}\left[I_2\left(q_r\left(\mathbf{H}_2\right)\right)\right] \\ &-\lambda_2\left(P_R - \mathbb{E}_{\mathbf{H}_2\in\mathbf{H}_P}\left[q_r\left(\mathbf{H}_2\right)\right]\right). \end{aligned} \quad (31)$$

Differentiating this Lagrangian we obtain

$$\frac{\partial\mathcal{L}\left(q_r\left(\mathbf{H}_2\right),\lambda_2\right)}{\partial q_r\left(\mathbf{H}_2\right)} = \begin{cases} \lambda_2 - \frac{\partial I_2(q_r(\mathbf{H}_2))}{\partial q_r(\mathbf{H}_2)}, & \mathbf{H}_2 \in \mathbf{H}_P, \\ 0, & \mathbf{H}_2 \in \mathbf{H}_N. \end{cases} \quad (32)$$

By equating the gradient of the above Lagrangian to zero and solve for $q_r\left(\mathbf{H}_2\right)$, we obtain the closed-form optimal solution expressed as

$$q_r^*\left(\mathbf{H}_2\right) = \begin{cases} \left(\frac{-d_2+\sqrt{d_2^2-4c_2\left(e_2-\frac{a_2}{\ln(2)\lambda_2^*}\right)}}{2c_2}\right)^+, & a_2 > 0, \\ 0, & a_2 \leq 0. \end{cases} \quad (33)$$

where $\lambda_2^* > 0$ is a unique constant satisfying

$$\mathbb{E}_{\mathbf{H}_2}[q_r^*(\mathbf{H}_2)] = P_R, \quad (34)$$

and it can be found by bisection method. The MRC-based pre-coding vector at the relay for the NAF system is finally expressed as

$$\mathbf{w}_{NAF}^* = \left(\sqrt{\frac{\rho q_r^*\left(\mathbf{H}_2\right)}{\rho q_1\alpha^2+\alpha}}\right)\bar{\mathbf{h}}_{SR}. \quad (35)$$

IV. Illustrative Results

In this section, numerical results are provided to illustrate the advantages in terms of the achievable rate of the proposed AF systems over the previously considered AF systems. In all simulation results, the achievable rate is calcualted over 10^7 channel realizations and the channel gains are assumed to be independent zero-mean complex circularly symmetric Gaussian as $h_{SD}, h_{SR_n}, h_{RD_n} \sim \mathcal{CN}(0,1)$ with $n = 1, ...N$. To demonstrate the gains offered by using the considered OAF and NAF systems with the proposed MRC-based relay pre-coding scheme, we compare the following systems:

- OAF-n, NAF-n, ($n \geq 2$): the OAF and NAF systems with n antennas at the relay using MRC-based relay pre-coding scheme.
- 2-Hop-n, ($n \geq 2$): the 2-Hop AF system with n antennas at the relay using the optimal relay pre-coding matrix proposed in [17].
- OAF-1, NAF-1: the single-antenna OAF and NAF systems using the optimal relay power adaptation schemes in [11].

To have a fair comparison, the transmit power at the source in each cooperative frame is $q = 1$ for OAF and 2-Hop systems. On the other hand, for NAF systems, the power allocated at the source in the broadcasting and cooperative phases are $q_1 = 0.5$ and $q_2 = 0.5$, respectively.

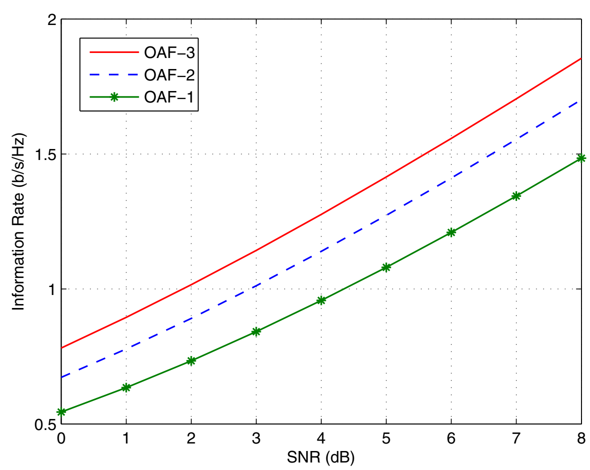

Fig. 2. The achievable rates of cooperative OAF systems having one, two, and three antennas at the relay, respectively.

At first, to demonstrate the benefit of having multiple antenna at the relays, Fig. 2 shows the achievable rates of three OAF systems using one, two, and three antennas at the relay, respectively. Observe from Fig. 2 that the achievable rate significantly increases as the number of antennas at the relay increases. Specifically, the OAF-3 system provides about 1 dB gain over the OAF-2 system and about 2.5 dB gain over the OAF-1 system in a wide range of SNRs.

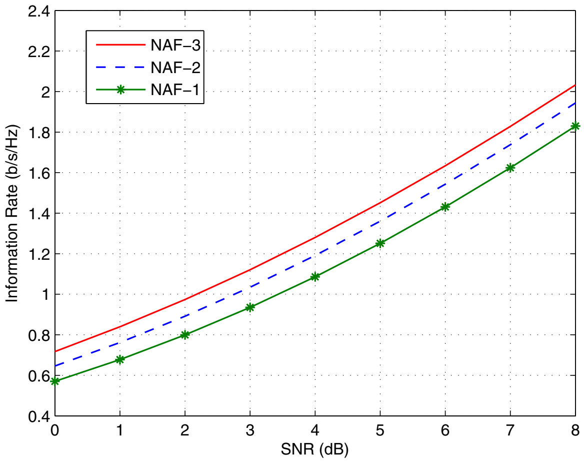

Fig. 3. The achievable rates of cooperative NAF systems having one, two, and three antennas at the relay, respectively.

Similar behavior can be observed in the NAF systems as shown in Fig. 3. In particular, the gain of the NAF-3 system is about 0.45dB and 1.2dB over the NAF-2 and NAF-1 systems, respectively.

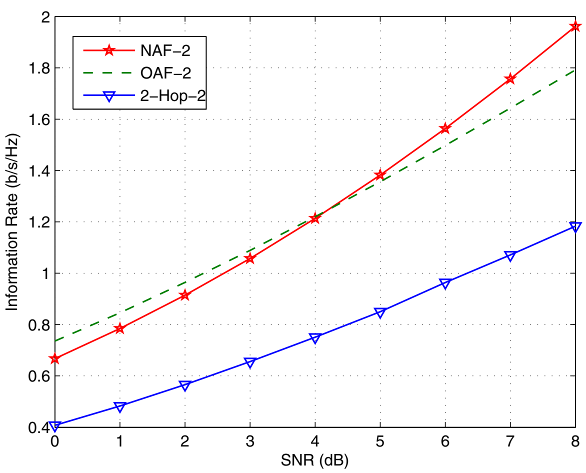

Fig. 4. Rate comparison between OAF, NAF, and 2-hop AF systems.

Finally, Fig. 4 compares the achievable rates achieved by cooperating AF relaying using the OAF and NAF protocols and that achieved by the 2-hop AF system. We only consider a relay having two antennas. Similar behavior can be observed for any number of antennas at the relay. It can be seen from Fig. 4 that both OAF and NAF protocols provide significantly rate improvement over the dual-hop system. While the OAF-n systems perform better than the NAF counterpart in low SNR regions, the NAF protocol becomes superior at sufficiently high SNRs.

V. Conclusions

In this paper, we proposed an optimal MRC-based precoding technique to exploit full CSI at a multi-antenna relay in cooperative OAF and NAF relay systems. The optimal MRC-based relay pre-coding vector was derived via the closed-form optimal solution of the power amplification factor at the relay. Numerical results demonstrated that the proposed scheme can improve the achievable rate significantly over both the conventional dual-hop AF multi-antenna systems and the cooperative AF single-antenna systems.

References

[1] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, pp. 74–80, Oct. 2004.

[2] R. U. Nabar and H. Bölcskei and F. W. Kneubühler, "Fading relay channel: Performance Limits and Space-Time Signal Design," *IEEE J. Sel. Areas in Commun.*, vol. 22, pp. 1099–1109, Aug. 2004.

[3] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, pp. 3037–3063, Sept. 2005.

[4] B. Wang, J. Zhang, and A. Host-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inform. Theory*, vol. 51, pp. 29–43, Jan. 2005.

[5] Y. Ding, J. Zhang, and K. M. Wong, "Ergodic channel capacities for the amplify-and-forward half-duplex cooperative systems," *IEEE Trans. Inform. Theory*, vol. 55, pp. 713–730, Feb. 2009.

[6] A. Firag, P. Smith, and M. McKay, "Capacity analysis for MIMO two-hop amplify-and-foward relaying systems with the source to destination link," *Proc. IEEE Int. Conf. Commun.*, pp. 1–6, June 2009.

[7] S. Jin, M. McKay, C. Zhong, and K.-K. Wong, "Ergodic capacity analysis of amplify-and-forward MIMO dual-hop systems," *IEEE Trans. Inform. Theory*, vol. 56, pp. 2204–2224, May 2010.

[8] M. Hasna and M.-S. Alouini, "End-to-end Performance of Transmission Systems with Relays over Rayleigh-Fading Channels," *IEEE Trans. on Wireless Commun.*, vol. 2, pp. 1126–1131, Nov. 2003.

[9] M. Hasna and M.-S. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh-Fading channels," *IEEE Trans. on Wireless Commun.*, vol. 3, pp. 1999–2004, Nov. 2004.

[10] L. J. Rodriguez, A. Helmy, N. H. Tran, and T. Le-Ngoc, "Optimal power adaption for NAF relaying with channel side information," *Proc. IEEE Int. Conf. Commun.*, pp. 4537–4541, June 2012.

[11] L. J. Rodriguez, N. H. Tran, A. Helmy, and T. Le-Ngoc, "Optimal power adaption for cooperative AF relaying with channel side information," *IEEE Trans. Veh. Technol.*, vol. 62, pp. 3164–3174, Sept. 2013.

[12] T. X. Tran, N. H. Tran, H. R. Bahrami, H. Dinh, and S. Sastry, "On Achievable Rate and Ergodic Capacity of OAF Multiple-Relay Networks with CSI," *Proc. IEEE Veh. Technol. Conf.*, June 2013.

[13] T. X. Tran, N. H. Tran, and H. R. Bahrami, "Optimal Power Sharing Strategies in NAF Multiple-Relay Networks with CSI," *Proc. IEEE Int. Conf. Commun.*, pp. 3213–3217, June 2013.

[14] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[15] K. Azarian, H. E. Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inform. Theory*, vol. 51, pp. 4152–4172, Dec. 2005.

[16] L. Sanguinetti, Antonio A. D'Amico and Y. Rong, "A Tutorial on the Optimization of Amplify-and-Forward MIMO Relay Networks," *IEEE J. Sel. Areas Commun.*, vol. 30, pp. 1331 – 1346, Sept. 2012.

[17] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Trans. on Wireless Commun.*, vol. 6, no. 4, pp. 1398–1407, 2007.