On Achievable Rate and Ergodic Capacity of OAF Multiple-Relay Networks with CSI

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***Abstract*—This paper investigates the achievable rate and ergodic capacity of an orthogonal amplify-and-forward (OAF) half-duplex multiple-relay network with direct link where multiple relays use channel state information (CSI) to cooperate with the source and destination. The relays are subject to two types of power constraint: the total average power constraint (TAPC) and the individual average power constraint (IAPC). In the first step, by assuming a fixed input covariance matrix at the source, we derive an optimal power allocation (OPA) scheme among the relays via optimal instantaneous power amplification coefficients to maximize the achievable rate. The closed-form optimal solutions are obtained for the considered system under either the TAPC or both the TAPC and IAPC. Next, we derive the ergodic capacity by jointly optimizing the input covariance matrix and the power allocation at the relays. We show that this is a bi-level non-convex problem and solve this using Tammer decomposition method. This approach allows us to convert the original optimization problem to a master problem and a set of sub-problems that have closed-form solutions as obtained in the first step. The ergodic capacity is then obtained using an iterative water-filling-based algorithm.**

***Index Terms*—Channel state information, distributed water-filling, ergodic capacity, multiple relays, orthogonal amplify-and-forward, power adaptation, relay channel.**

I. Introduction

Cooperative relaying is an effective approach to enhance the reliability and data-rate of wireless channels [1]–[4]. Here, network nodes assist each other by relaying transmissions. Among different cooperative relaying protocols, the amplify-and-forward (AF) scheme is popular because of its performance and the ease of implementation [4]–[9]. In this scheme, an intermediate node (relay) receives a signal from the source and transmits an amplified, perhaps noisy, version of that signal toward the destination.

Recent investigations on AF networks have focused on the AF schemes that consider the direct link to fully exploit the degrees of freedom offered by relay channels [2], [4], [7]. In these approaches, a relay can amplify the received signal using a suitable amplification coefficient that depends on the channel state information (CSI) at the relay. When a relay could only obtain the channel distribution information (CDI) of the source-relay channel, the power-constrained capacity of an ergodic non-orthogonal AF (NAF) channel was reported in [7] over fading channels. On the other hand, when a relay has an instantaneous knowledge of the source-relay channel, the channel inversion (CI) technique can be used to amplify the received signal at a desired power level [3], [5]. Recently, it was shown in [10], [11] that a relay could adapt its instantaneous transmitted power to improve the data-rate over what could be achieved using CDI and CI if the relay could cooperate with the destination to acquire complete CSI. However, this work only studied the single relay system with a fixed covariance matrix, and hence, only the achievable rate could be obtained.

Extending this idea, i.e., adapting instantaneous power at the relay nodes by exploiting the complete CSI, to a multiple-relay system presents many challenges. Often, in practical systems, there is a maximum power budget that constrains the nodes in the system and the local power at each relay may be constrained. It is necessary to account for these constraints and determine a suitable scheme to share power among the nodes. For these reasons, the approach in this paper considers both a *Total Average Power Constraint* (TAPC) and an *Individual Average Power Constraint* (IAPC) when addressing optimal power allocation (OPA) among the relay nodes. Further, to achieve the capacity of the channel, we jointly optimize the input covariance matrix and the power sharing strategy among the relays.

This paper presents new results for the achievable rate and ergodic capacity of an AF half-duplex multiple-relay network with the direct link where the relays exploit complete CSI to cooperate with the source and destination. We focus on the orthogonal AF (OAF) protocol [2], but the results can be extended to other AF protocols that have a source-destination link. We first generalize the achievable rate results in [10], [11] (for single-relay system) to a system with multiple relays. Specifically, for a given input covariance matrix, we identify OPA strategies among the relays by means of optimal instantaneous power amplifications that are necessary to maximize the achievable rate. We present closed-form solutions for systems under the TAPC and under both the TAPC and IAPC. These solutions are then used to address the capacity limit of the considered system. In particular, the optimal covariance matrix at the source is jointly optimized with the OPA at the relays to achieve the ergodic capacity. The optimization problem in this step is a bi-level non-convex problem for which it is not feasible to derive a closed-form solution. Hence, we utilize the Tammer decomposition method [12] to transform the original problem to an equivalent master problem and a set of sub-problems having

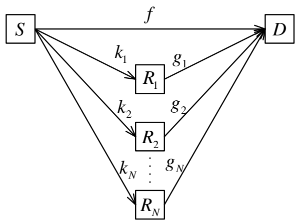

Fig. 1. The half-duplex multiple-relay OAF system.

closed-form solution obtained in the first step. The ergodic capacity can then be obtained via an iterative water-filling-based algorithm. Finally, numerical results are provided to confirm our analysis.

II. System Model

A. OAF protocol

The half-duplex AF system shown in Fig. 1 consists of N relays that assist transmissions between a source (S) and a destination (D). We consider the OAF transmission protocol in which the source and relays use separate time slots to transmit signals [2]. The transmission is carried out in multiple cooperative frames following the protocol proposed in [5], [13]. In particular, a cooperative frame consists of N sub-frames, each divided into two phases, namely the broadcasting phase and cooperative phase. In the nth sub-frame, $1 \leq n \leq N$, only relay R_n is active while other relays remain silent. In the broadcasting phase of the nth sub-frame, S sends the signal x_n to both R_n and D. The received signals at R_n and D can be written as

$$r_n = \sqrt{E_S} k_n x_n + w_n,$$
$$y_{n1} = \sqrt{E_S} f x_n + v_{n1},$$

where E_S is a constant related to the transmitted power; w_n, v_{n1} are the i.i.d zero-mean circularly Gaussian noises with variance N_0, denoted as $\mathcal{CN}(0, N_0)$; k_n and f are the instantaneous $S - R_n$ and $S - D$ complex channel gains, respectively, within a current frame. In the cooperative phase of the nth sub-frame, R_n forwards the amplified noisy version of the signal it received during the broadcasting phase to D while S remains silent. The received signal at D in this phase is represented as

$$y_{n2} = g_n b_n (\sqrt{E_S} k_n x_n + w_n) + v_{n2},$$

where $v_{n2} \sim \mathcal{CN}(0, N_0)$, b_n is the amplification coefficient at the relay R_n, and g_n is the instantaneous $R_n - D$ channel gain. It should be noted that in a practical system, a relay node can engage in other transmissions when its relaying function is inactive. This multiple-relay protocol has advantage in terms of diversity and rate performances as shown in [5], [13]. The instantaneous complex channel gains f, k_n, and g_n with $n = 1, ...N$ are assumed to remain unchanged during at least one cooperative frame [7].

By whitening the noise component, the input-output relation of the OAF system can be written in matrix form as

$$\mathbf{y} = \sqrt{E_S} \mathbf{H} \mathbf{x} + \mathbf{n}. \tag{1}$$

In (1), $\mathbf{x} = [x_1, x_2, ..., x_N]^T$ is the input vector, $\mathbf{y} = [y_{11}, y_{12}, ..., y_{N1}, y_{N2}]^T$ is the output vector, and $\mathbf{H}$ is a $2N \times N$ matrix given by $\mathbf{H} = diag(\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_N)$, with $\mathbf{H}_n = \begin{bmatrix} f \\ \alpha_n b_n k_n g_n \end{bmatrix}$, $n = 1, ...N$. Furthermore, $\alpha_n = \frac{1}{\sqrt{1 + b_n^2 |g_n|^2}}$ is the noise whitening factor.

B. Channel State Information (CSI) and Power Constraints

The channels between different nodes in the system are assumed to be independently Rayleigh distributed, i.e., $f \sim \mathcal{CN}(0, \phi_f)$, $k_n \sim \mathcal{CN}(0, \phi_{k_n})$ and $g_n \sim \mathcal{CN}(0, \phi_{g_n})$. It is assumed that D has perfect CSI [4], [5], [7]–[9] and such information can also be made available at each relay using a feedback channel from the destination [10], [11].

Suppose that the input signal uses Gaussian codebook and S has no CSI knowledge; it has been shown in [7] that the optimal input covariance matrix will be the diagonal matrix given as $\mathbf{Q} = diag(q_1, q_2, ..., q_N)$. Hence, $q_n E_S$ can be interpreted as the average power of the source in the nth sub-frame. The average transmission power constraint at the source can then be expressed as $tr(\mathbf{Q}) = \sum_{n=1}^{N} q_n \leq P_S$. Furthermore, let $\mu_n E_S$ be the *instantaneous* power allocated to the relay R_n in one cooperative frame. Under the assumption of full CSI knowledge at the relays, the relays can use this information to adjust their instantaneous transmit powers in order to adapt to the current channel conditions.

For convenience, let

$$\mathbf{h} = [f, k_1, ...k_N, g_1, ...g_N]^T.$$

Then, the instantaneous value of μ_n depends on $\mathbf{h}$, i.e., $\mu_n = \mu_n(\mathbf{h})$. Such a variation is reflected in the use of an instantaneous power amplification coefficient at each relay R_n which can be represented as

$$b_n = \sqrt{\frac{\mu_n(\mathbf{h})\rho}{|k_n|^2 q_n \rho + 1}}, \tag{2}$$

where $\rho = E_S/N_0$ is the normalized SNR. It is assumed that the total average power available for all relays is limited by P, i.e.,

$$\sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P, \tag{3}$$

where $f(\mathbf{h})$ is the probability density function of the channel state $\mathbf{h}$. Under a more realistic scenario, each relay can also have its own average power constraint limited by a fixed P_n due to the constraint on each individual RF chain. Such a constraint can be expressed as

$$\int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P_n. \tag{4}$$

It is also assumed that $\sum_{n=1}^{N} P_n \geq P$; otherwise the IAPC will imply the TAPC. Finally, it should be noted that when the relays only have knowledge of the channel distribution information (CDI) of the S-R_n link, the amplification coefficient at relay R_n is [2], [7], [14]:

$$b_n^{(CDI)} = \sqrt{\frac{P\rho}{N\left(\mathbb{E}[|k_n|^2]q_n\rho + 1\right)}}. \tag{5}$$

On the other hand, when only the CSI of the S-R_n link is available at the relays, each relay R_n can use the channel inversion with the following amplification coefficient [3], [5]:

$$b_n^{(CI)} = \sqrt{\frac{P\rho}{N\left(|k_n|^2 q_n\rho + 1\right)}}. \tag{6}$$

C. Achievable Rate and Ergodic Capacity

From (1), the instantaneous mutual information between the input and output given CSI knowledge at the destination can be expressed as

$$I(\mathbf{x}, \mathbf{y}|\mathbf{h}) = \log\det[\mathbf{I}_N + \rho\mathbf{H}^{\dagger}\mathbf{H}\mathbf{Q}], \tag{7}$$

where † denotes the Hermitian operator and $log(.)$ is the base-2 logarithm. Therefore, for a given input covariance matrix $\mathbf{Q}$, the maximum achievable rate under the OAF protocol can be defined as

$$R(\mathbf{Q}) = \max_{\mu_n(\mathbf{h}), n=\overline{1,N}} \int_{\mathbf{h}} I(\mathbf{x}, \mathbf{y}|\mathbf{h}) f(\mathbf{h})\, d\mathbf{h}. \tag{8}$$

In (8), the maximum is taken over the feasible set of the instantaneous power allocation at the relays $\mu_n(\mathbf{h})$. By further optimizing the input covariance matrix at the source and the power allocation at the relays simultaneously, one can achieve the ergodic capacity of the system which can be expressed as

$$C = \max_{\substack{q_n > 0, tr(\mathbf{Q}) \leq P_S, \\ \mu_n(\mathbf{h}), n=\overline{1,N}}} \int_{\mathbf{h}} I(\mathbf{x}, \mathbf{y}|\mathbf{h}) f(\mathbf{h})\, d\mathbf{h}, \tag{9}$$

where the maximum is taken over the feasible set of $\mathbf{Q}$ and $\mu_n(\mathbf{h})$.

III. Achievable Rate and Optimal Power Allocation at the Relays

In this section, we shall develop the optimal power sharing schemes among the relays to maximize the achievable rate $R(\mathbf{Q})$. We first consider the TAPC, and then both the TAPC and IAPC simultaneously. For convenience, let $x_n = |g_n|^2$, $y = |f|^2$, $z_n = |k_n|^2$. Using (7), we obtain

$$I(\mathbf{x}, \mathbf{y}|\mathbf{h}) = \sum_{n=1}^{N} \log\left(\Gamma_n[\mu_n(\mathbf{h})]\right) = \sum_{n=1}^{N} \log\left(\frac{\Gamma_{n1}[\mu_n(\mathbf{h})]}{\Gamma_{n2}[\mu_n(\mathbf{h})]}\right), \tag{10}$$

where

$$\Gamma_{n1}[\mu_n(\mathbf{h})] = \mu_n(\mathbf{h})\left(\rho q_n y + \rho q_n z_n + 1\right)\rho x_n + \left(\rho^2 q_n^2 z_n y + \rho q_n z_n + \rho q_n y + 1\right),$$

and $\Gamma_{n2}[\mu_n(\mathbf{h})] = \mu_n(\mathbf{h}) x_n + (\rho q_n z_n + 1)$.

A. Total Average Power Constraint

Suppose now that the TAPC in (3) is imposed on all N relays. The set of optimal instantaneous powers allocated at the relays $\{\mu_n^*(\mathbf{h})\}$ that maximizes $R(\mathbf{Q})$ in (8) can be obtained by solving the following optimization problem

$$\max_{\substack{\mu_n(\mathbf{h}) \geq 0 \\ n=\overline{1,N}}} \int_{\mathbf{h}} I(\mathbf{x}, \mathbf{y}|\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \quad s.t \quad \sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P. \tag{11}$$

Let $I(\mathbf{x}, \mathbf{y}|\mathbf{h}) = I(\boldsymbol{\mu}(\mathbf{h}))$ where $\boldsymbol{\mu}(\mathbf{h}) = [\mu_1(\mathbf{h}), \ldots, \mu_N(\mathbf{h})]$. Differentiating the objective function in (11) with respect to $\mu_n(\mathbf{h})$, we have

$$\begin{aligned}
&\frac{\partial}{\partial\mu_n(\mathbf{h})} \int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} = \frac{a_n}{\ln(2)[c_n\mu_n^2(\mathbf{h}) + d_n\mu_n(\mathbf{h}) + e_n]} f(\mathbf{h}), \\
&\frac{\partial^2}{\partial\mu_n^2(\mathbf{h})} \int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} = \frac{-a_n[2c_n\mu_n(\mathbf{h}) + d_n]}{\ln(2)[c_n\mu_n^2(\mathbf{h}) + d_n\mu_n(\mathbf{h}) + e_n]^2} f(\mathbf{h}), \\
&\frac{\partial^2}{\partial\mu_m(\mathbf{h})\partial\mu_n(\mathbf{h})} \int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} = 0, \ (m \neq n),
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
a_n &= \rho^2 q_n z_n x_n(\rho q_n z_n + 1), \\
c_n &= \rho^2 x_n^2(\rho q_n y + \rho q_n z_n + 1), \\
d_n &= \rho x_n(\rho q_n z_n + 1)(2\rho q_n y + \rho q_n z_n + 2), \\
e_n &= (\rho q_n y + 1)(\rho q_n z_n + 1)^2.
\end{aligned} \tag{13}$$

It can be seen that the Hessian of the objective function in (11) is diagonal with the strictly negative elements. Hence, we can verify that the problem in (11) is *concave* and the solution in the form of $\mu_n(\mathbf{h}) = \mu_n^*(\mathbf{h})$ is *globally optimal*. The problem in (11) can then be solved by using the Karush-Kuhn-Tucker (KKT) conditions. In particular, the optimal instantaneous power allocation $\mu_n^*(\mathbf{h})$ for the OAF system under the TAPC is obtained as

$$\mu_n^*(\mathbf{h}) = \left(\frac{-d_n + \sqrt{d_n^2 - 4c_n\left(e_n - \frac{a_n}{\lambda^*\ln(2)}\right)}}{2c_n}\right)^+, \tag{14}$$

where $(x)^+ = max(0, x)$, $\lambda^* > 0$ is a unique constant that satisfies the TAPC in (3) and it can be found numerically.

B. Incorporating Individual Average Power Constraint

Now we consider the OAF system under both the TAPC and IAPC. The optimization problem in (11) will become

$$\max_{\substack{\mu_n(\mathbf{h}) \geq 0 \\ n=\overline{1,N}}} \int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} \ s.t. \begin{cases} \sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P, \\ \int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P_n. \end{cases} \tag{15}$$

It is easy to verify that the problem in (15) is also *concave* and the *globally optimal* solution, $\mu_n^*(\mathbf{h})$, can be obtained by using the KKT conditions. The optimal instantaneous power allocation for the OAF system under both the TAPC and the IAPC can then be expressed as

$$\mu_n^*(\mathbf{h}) = \left(\frac{-d_n + \sqrt{d_n^2 - 4c_n\left(e_n - \frac{a_n}{(\lambda^* + \lambda_n^*)\ln(2)}\right)}}{2c_n}\right)^+, \tag{16}$$

where $\lambda^* > 0$ and $\lambda_n^* \geq$ are the constants satisfying

$$\begin{cases} \sum\limits_{n=1}^{N} \int\limits_{\mathbf{h}} \mu_n^*(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} = P, \\ \int\limits_{\mathbf{h}} \mu_n^*(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P_n, \\ \lambda_n^* \left(P_n - \int\limits_{\mathbf{h}} \mu_n^*(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \right) = 0. \end{cases} \tag{17}$$

The two parameters λ^* and λ_n^* depend on the channel distribution $f(\mathbf{h})$, the SNR ρ, and the average power constraints P and $\{P_n\}$, and they can be found numerically.

IV. Ergodic Capacity of the OAF system

In this section, we present the ergodic capacity of the system by simultaneously optimizing the diagonal input covariance matrix at the source and the instantaneous power allocations at the relays. As in the previous section, we consider the TAPC, and both the TAPC and IAPC.

A. Total Average Power Constraint

Under the TAPC, the ergodic capacity of the OAF system can be defined as

$$\begin{aligned} C = &\max_{\substack{\mu_n(\mathbf{h}) \geq 0,\, q_n > 0 \\ n = \overline{1,N}}} \int\limits_{\mathbf{h}} I(\mathbf{x}, \mathbf{y}|\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \\ s.t. \quad &\sum_{n=1}^{N} \int\limits_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P, \sum_{n=1}^{N} q_n \leq P_S, \end{aligned} \tag{18}$$

where P_S is the total average power allocated to S in the broadcasting phase. In general, it is not possible to obtain the closed-form solution of (18) due to the complexity of the problem. As an alternative, by decomposing the problem into a set of sub-problems and making use of the closed-form solutions obtained in the previous section, we show that an optimal solution can be achieved by performing an iterative water-filling algorithm. For convenience, let

$$\Lambda_n(q_n, \mu_n) = \int\limits_{\mathbf{h}} \log(\Gamma_n[\mu_n(\mathbf{h})]) f(\mathbf{h})\, d\mathbf{h}.$$

Then from (10), we can rewrite the objective function in (18) as follows

$$\begin{aligned} \int\limits_{\mathbf{h}} I(\mathbf{x}, \mathbf{y}|\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} &= \sum_{n=1}^{N} \int\limits_{\mathbf{h}} \log(\Gamma_n[\mu_n(\mathbf{h})]) f(\mathbf{h})\, d\mathbf{h} \\ &= \sum_{n=1}^{N} \Lambda_n(q_n, \mu_n). \end{aligned} \tag{19}$$

Let $\mathbf{q_S} = (q_1, q_2, ..., q_N)$. Observe from (18) and (19) that the objective function exhibits a block sparse structure in which we can consider μ_n as the local variable that appears only in one block and $\mathbf{q_S}$ as the global variable that appears in all the blocks. Exploiting this property, we effectively utilize the Tammer decomposition method [12] and transform the original problem into an equivalent master problem and a set of sub-problems. The key idea is to solve the sub-problems using the closed-form solutions in (14) and then use the information obtained from their solutions to determine the optimal value of the global variable. In Tammer decomposition method, solving the problem in (18) is equivalent to solving the following master problem

$$\max_{\substack{q_n > 0 \\ n = \overline{1,N}}} \sum_{n=1}^{N} \Lambda_n^*(q_n) \quad s.t. \quad \sum_{n=1}^{N} q_n \leq P_S, \tag{20}$$

where $\Lambda_n^*(q_n)$ is the optimal-value function corresponding to the nth sub-problem and it can be written as

$$\Lambda_n^*(q_n) = \max_{\mu_n(\mathbf{h}) \geq 0} \Lambda_n(q_n, \mu_n) \;\; s.t. \;\; \sum_{n=1}^{N} \int\limits_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P. \tag{21}$$

Notice that all the sub-problems are subject to a global constraint. Therefore, instead of solving the sub-problems separately as in [12], we need to solve them simultaneously. These sub-problems can be in fact obtained by setting $\mathbf{q_S}$ to a fixed value. It is because each sub-problem in (21) has the same solution as the problem in (11); the solution to this problem is shown in (14).

To solve the master problem, a line search method can then be applied. At each iteration, all the sub-problems are solved to determine the new search direction and, therefore, the value of $\mathbf{q_S}$ will be adjusted. Note that the sub-problem in (21) is feasible for all value of q_n satisfying $q_n > 0$ and $\sum\limits_{n=1}^{N} q_n \leq P_S$. Since the feasibility of the sub-problem is always ensured during the line search, it has been shown in [12] that there exists an equivalent master problem maximizer. To facilitate the line search implementation, we can approximate the master problem in (20) by using a penalty function M_k in which $\lim\limits_{k \to \infty} M_k \to \infty$ as follows

$$\max_{\substack{q_n > 0 \\ n = \overline{1,N}}} \left[\sum_{n=1}^{N} \Lambda_n^*(q_n) - M_k \left(P_S - \sum_{n=1}^{N} q_n \right)^2 \right]. \tag{22}$$

Let $\Lambda^*(\mathbf{q_S}) = \sum\limits_{n=1}^{N} \Lambda_n^*(q_n) - M_k \left(P_S - \sum\limits_{n=1}^{N} q_n \right)^2$ in which the optimal-value function $\Lambda_n^*(q_n)$ is obtained by solving the sub-problems with the closed-form solutions given in (14). The iterative algorithm, which is referred to as MASSOL algorithm, to solve the master problem with a sufficiently large M_k can then be summarized as follows:

MASSOL Algorithm

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(1) **(Initialization)** - Initialize the optimality tolerance $\epsilon := \epsilon_0$.

(2) **(Starting point)** - Set $\mathbf{q_s} := \mathbf{q_{s_0}}$ with $\mathbf{q_{s_0}}$ be an initial point. Solve the sub-problems in (21) to obtain $\mathbf{\Lambda}^*(\mathbf{q_{S_0}})$ and $\nabla \mathbf{\Lambda}^*(\mathbf{q_{S_0}})$.

(3) **(Line search)** - Generate the descent direction p. Set the initial step length $\tau := 1$ and $\breve{\mathbf{q}}_\mathbf{S} = \mathbf{q_S} + \tau p$. Solve the sub-problems in (21) to verify the sufficient decrease condition (SDC) and adjust the step length τ until we find $\breve{\mathbf{q}}_\mathbf{S}$ that satisfies SDC. Set $\mathbf{q_S} = \breve{\mathbf{q}}_\mathbf{S}$.

(4) **(Convergence check)** - Repeat step (3) if $\|\nabla \mathbf{\Lambda}^*(\mathbf{q_S})\| > \varepsilon_0$. Otherwise, terminate the line search.

B. Incorporating Individual Average Power Constraint

Under both the IAPC and TAPC, the ergodic capacity of the OAF system can be expressed as

$$C = \max_{\substack{\mu_n(\mathbf{h})\geq 0,\, q_n>0 \\ n=\overline{1,N}}} \int_{\mathbf{h}} I\left(\mathbf{x}, \mathbf{y}|\mathbf{h}\right) f\left(\mathbf{h}\right) d\mathbf{h}$$

$$s.t. \begin{cases} \int_{\mathbf{h}} \mu_n\left(\mathbf{h}\right) f\left(\mathbf{h}\right) d\mathbf{h} \leq P_n, \\ \sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n\left(\mathbf{h}\right) f\left(\mathbf{h}\right) d\mathbf{h} \leq P, \sum_{n=1}^{N} q_n \leq P_S. \end{cases} \quad (23)$$

The above problem can be decomposed into the master problem as in (20) and the set of N sub-problems, each can be written as

$$\Lambda_n^*\left(q_n\right) = \max_{\mu_n(\mathbf{h})\geq 0} \Lambda_n\left(q_n, \mu_n\right)$$

$$s.t. \quad \int_{\mathbf{h}} \mu_n\left(\mathbf{h}\right) f\left(\mathbf{h}\right) d\mathbf{h} \leq P_n, \sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n\left(\mathbf{h}\right) f\left(\mathbf{h}\right) d\mathbf{h} \leq P. \quad (24)$$

It can be shown that the set of N sub-problems given in (24) have the same solutions as the problem in (15). The master problem can then be solved by the MASSOL algorithm in which the sub-problems have the closed-form solutions in (16).

V. Illustrative Results

In this section, we present numerical results to illustrate the achievable rate and ergodic capacity derived in the previous sections. For the brevity of the presentation, we evaluate systems that have two relays. Similar behaviour is also observed in systems with more than two relays. We adopt the distance-dependent path-loss model [15] such that the channel variances depend on the path-loss exponents, i.e., $\phi_f = d_f^{-v}, \phi_{k_n} = d_{k_n}^{-v}, \phi_{g_n} = d_{g_n}^{-v}$, where v is the path-loss exponent and is set to 3 in our simulations. Due to the space limitation, we only present results for the asymmetric configuration in which the channel parameters are set as $d_f = 1, d_{k_1} = d_{g_1} = 2, d_{k_2} = d_{g_2} = 0.5, P_S = P = 4, P_n = 0.7P$. Similar results can also be obtained for other channels. In all these results, the SNR is defined as $\rho = E_S/N_0$.

A. Achievable Rate

This subsection illustrates the gain in terms of achievable rate provided by our OPA scheme at the relays using full CSI. For comparison, we also evaluate and consider the two conventional systems using CDI and CI as a benchmark. For all systems under consideration, equal power allocation at S is assumed, i.e., all diagonal elements of the input covariance matrix are identical.

Fig. 2 shows the achievable rate of the CSI, CDI and CI systems under the same TAPC. Observe from Fig. 2 that by exploiting CSI, a gain of around 0.6dB can be achieved over the CDI and CI systems in a wide range of SNRs. We also observe that the CDI system is slightly better than the CI system at low SNRs. However, the difference between the CDI and CI systems is negligible.

Fig. 3 compares the achievable rate of CSI system under the TAPC and under both the TAPC and IAPC. It can be seen that there is a penalty by imposing individual power constraint on each relay. The loss is of about 0.4dB over various SNR regions. However, the system with CSI under both the TAPC and IAPC still outperforms the two CDI and CI systems under the TAPC only.

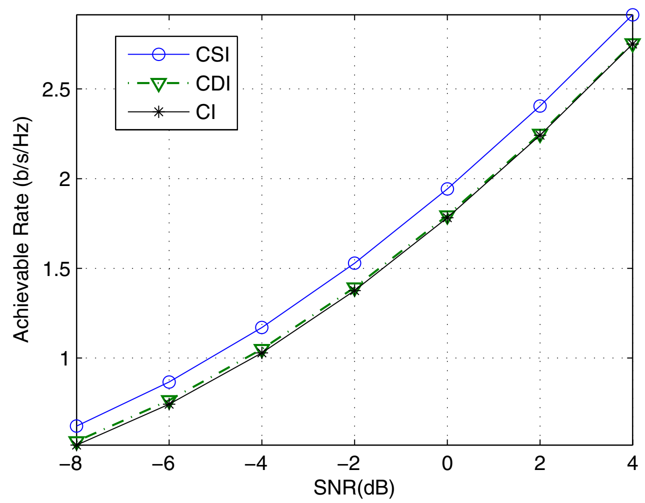

Fig. 2. The achievable rate of the OAF system with different power allocation schemes at the relays under TAPC.

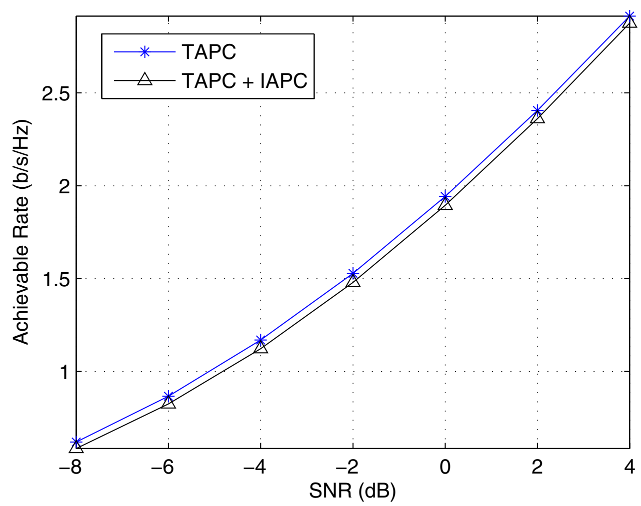

Fig. 3. The achievable rate of the OAF system with CSI under TAPC and under both TAPC and IAPC.

B. Ergodic Capacity

In this subsection, we further examine the system's ergodic capacity by jointly considering the effect of the covariance matrix at the source and the OPA scheme at the relays. For comparison, besides the optimal system using the optimal covariance matrix at the source, which is denoted as OPA-S, we shall also consider two other power allocation schemes at the source. These include i) EPA-S - a system with equal power allocation at the source, i.e., $q_1 = q_2 = Ps/2$; and ii) [2:1] - a system with the ratio $q_1 : q_2 = 2 : 1$ at the source. Note that all systems employ the OPA scheme at

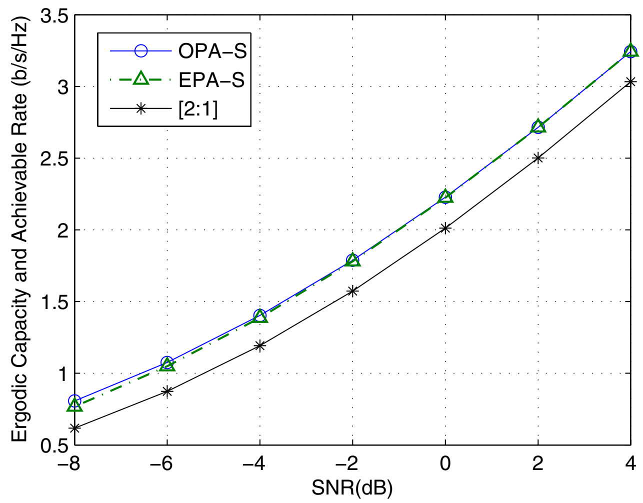

Fig. 4. The ergodic capacity and achievable rate of the OAF systems with different power allocation schemes at the source under the TAPC.

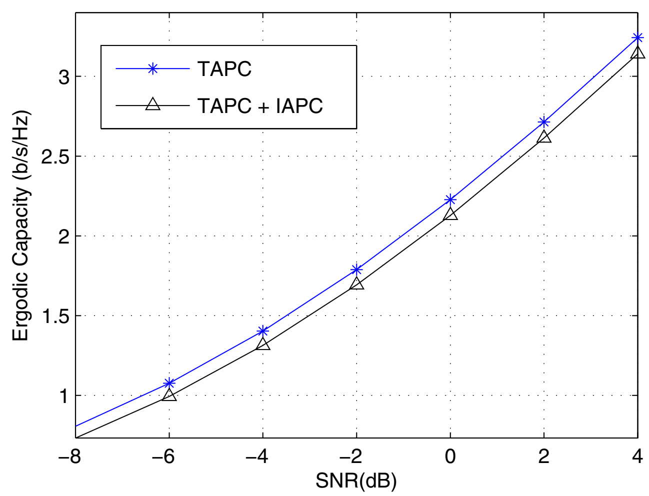

Fig. 5. The ergodic capacities of the OAF system under TAPC and under both the TAPC and IAPC.

the relays under the same TAPC. Fig. 4 plots the ergodic capacity and the rate achieved by the three schemes, OPA-S, EPA-S and [2:1], in a wide range of SNRs. It can be seen that at low SNRs, the OPA-S outperforms both the EPA-S and [2:1] systems. The gains over the EPA-S and [2:1] are around 0.2dB and 1dB, respectively. When SNR increases, the gap between the OPA-S and EPA-S becomes closer, and in fact, they almost coincide at sufficiently high SNRs. Although not explicitly shown here, our asymptotic analysis indicates that the EPA-S scheme achieves the capacity at sufficiently high SNRs.

Finally, Fig. 5 compares the ergodic capacity of the OPA-S systems under the TAPC and under both the TAPC and IAPC, respectively. Similar to the achievable rate, it can be seen that there is a penalty by imposing both the TAPC and IAPC at the relays. Specifically, the SNR gap between the system under both the TAPC and IAPC and that under only the TAPC is of about 0.35dB at various SNR values.

VI. Conclusions

This paper addressed the achievable rate and ergodic capacity of an OAF half-duplex multiple-relay network where multiple relays exploit full CSI of all links to cooperate with a pair of source and destination. By considering the TAPC and IAPC at the relays, optimal solutions for the OPA schemes in closed-form at the relays were first developed to achieve the maximum data rate for a given input covariance matrix. It was shown that the proposed OPA schemes with CSI at the relays significantly outperform the conventional systems using either CDI or CI. By further investigating the jointly optimal OPA at the relays and the optimal input covariance matrix at the source, we also established the ergodic capacity of the considered OAF system. The capacity could be evaluated using Tammer decomposition method via an iterative water-filling-based algorithm.

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