Optimal Power Sharing Strategies in NAF Multiple-Relay Networks with CSI

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***Abstract*—This paper addresses the problem of optimal power allocation among relaying nodes for a non-orthogonal amplify-and-forward (NAF) half-duplex relay network where multiple relays exploit channel state information (CSI) to cooperate with a pair of source and destination to maximize the source-destination mutual information (MI). In particular, assuming that the relays have complete CSI of all source-relay, source-destination, and relay-destination links, we investigate an optimal power sharing scheme via optimal power amplification coefficients at the relays. Given the nature of broadcasting and receiving collisions in NAF, the considered problem is non-convex. To overcome this drawback, we propose a novel method by evaluating the MI in different sub-domains of the vector channels. It is then demonstrated that the globally optimal solution can be obtained. In particular, the optimal solutions are derived in closed-form for the system under the total average power constraint (TAPC) and for the system under both TAPC and individual average power constraint (IAPC) at each relay. Numerical results are provided to quantify the significant gains offered by the proposed power sharing schemes over conventional schemes using either channel distribution information (CDI) or channel inversion (CI).**

***Index Terms*—Channel state information, non-orthogonal amplify-and-forward, power adaptation, relay channels.**

I. Introduction

Cooperative relaying [1]–[4] is an effective way to enhance the reliability and data rate over wireless channels and has been adopted in major wireless standards [5]. Among different cooperative relaying protocols, the amplify-and-forward (AF) scheme can be considered as one of the most popular protocols, due to its advantages in terms of both performance and implementation [6]–[9]. For AF relaying, an intermediate node (relay) receives a signal from the source and then simply transmits the noisy amplified version of that signal to the destination. Whilst there are different types of AF relaying, particular attention has been paid to the non-orthogonal AF (NAF) protocol proposed in [2]. It is because in NAF, the source-destination (S-D) link is used in both broadcasting and cooperative phases, which maximizes the degree of broadcasting and receiving collisions. In fact, the NAF protocol is considered to be favorable over all other AF schemes, not only from an information-theoretic point of view [6]–[9], but also in terms of diversity performance [10], [11].

In AF relaying, the amplification coefficient at the relay is chosen based on the availability of the channel information. For example, by assuming that a relay can only obtain the channel distribution information (CDI) of the source-relay (S-R) channel, the received signal at the relay is scaled by a constant-gain coefficient before being transmitted to the destination [2], [8], [10]. When the relay has an instantaneous knowledge of the S-R channel, the channel inversion (CI) technique can be used to amplify the received signal at a desired power level [3], [6]. Recently, motivated by the benefit offered by the cooperation between the relay and destination, references [12], [13] have examined an AF half-duplex relay system in which the relay acquires complete CSI of the S-D and S-R links via a feedback channel from the destination. It has been shown that with complete CSI, the relay could adapt its instantaneous transmitted power to improve the data rate and error performance over what could be achieved in the conventional AF relay systems. However, since [12], [13] only considered the single-relay system, a relay always has a fixed average power constraint and the optimal amplification coefficient is simply a water-filling process over the temporal domain. The extension of such a result to a multiple-relay system, which certainly provides more benefits, is important yet challenging. It is because in a coordinated multiple-relay network, one needs to take into account a power sharing scheme among multiple relay nodes for a given total average power constraint (TAPC). In addition, in practice, besides the global power constraint, each relay has its own individual average power constraint (IAPC), which makes the problem in hand more involved.

Motivated by the above discussions, this paper investigates the optimal power sharing strategies in a NAF half-duplex multiple-relay network with complete CSI at the relays. For the system under consideration, we shall use the mutual information (MI) with Gaussian inputs as a key measurement. Given that the NAF protocol is used, the considered optimization problem is not convex. Our method is to modify the problem by evaluating the MI in different sub-domains of the vector channels. It is then shown that the solution of the modified problem is still globally optimal. The method helps us establish in closed-form the optimal power adaptation schemes for the system with only the TAPC or with both the TAPC and IAPC. The system's performance in terms of MI is then addressed by numerical results to demonstrate its superiority over those using either CDI or CI.

II. System Model

A. NAF protocol

The half-duplex multiple-relay NAF system shown in Fig. 1 consists of one source (S), one destination (D) and N relay nodes ($R_1, R_2, ...R_N$). As proposed in [6], [14], signals are transmitted from S to D via a sequence of cooperative frames; each consists of two N-time-slot phases, the broadcasting phase and cooperative phase. The broadcasting phases are carried out in the $\{(2n-1)\}$th time slots with $1 \leq n \leq N$. For a given $(2n-1)$th time slot, only the relay R_n stays active and the source S sends the signal $x_{n,1}$ to both R_n and D. The received signals at R_n and D can be written respectively as

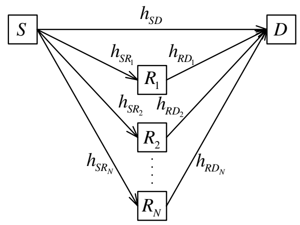

Fig. 1. The half-duplex multiple-relay NAF system.

$$r_n = \sqrt{E_s} h_{SR_n} x_{n,1} + w_n,$$
$$y_{2n-1} = \sqrt{E_s} h_{SD} x_{n,1} + v_{n,1},$$

where E_S is a constant related to the transmitted power; $w_n, v_{n,1}$ are the i.i.d zero-mean circularly Gaussian noises with variance N_0, denoted as $\mathcal{CN}(0, N_0)$; and h_{SR_n}, h_{SD} are the instantaneous S-R_n and S-D complex channel gains within a current cooperative frame, denoted as $\mathcal{CN}(0, \phi_{SR_n})$ and $\mathcal{CN}(0, \phi_{SD})$, respectively.

In NAF, the cooperative phases happen in the $\{2n\}$th time slots, $1 \leq n \leq N$ [2]. In particular, the source S transmits a new signal $x_{n,2}$ to D in the $(2n)$th time slot, while R_n forwards the amplified noisy version of the signal it received during the broadcasting phase to D. The received signal at D in the $(2n)$th time slot is given by

$$y_{2n} = \sqrt{E_s} h_{SD} x_{n,2} + h_{RD_n} b_n (\sqrt{E_s} h_{SR_n} x_{n,1} + w_n) + v_{n,2},$$

where $v_{n,2} \sim \mathcal{CN}(0, N_0)$, $h_{RD_n} \sim \mathcal{CN}(0, \phi_{RD_n})$, and b_n is the amplification coefficient at the relay R_n. Whitening the noise component, the NAF system can be written in matrix form as

$$\mathbf{y} = \sqrt{E_S}\mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where the input vector $\mathbf{x} = [x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}, ..., x_{N,1}, x_{N,2}]^T$ and the output vector $\mathbf{y} = [y_1, y_2, ..., y_{2N}]^T$. In (1), $\mathbf{H}$ is a $2N \times 2N$ block diagonal channel matrix given by $\mathbf{H} = diag(\mathbf{H}_1, \mathbf{H}_2, ..., \mathbf{H}_N)$ with $\mathbf{H}_n$, $n = 1, 2, ...N$ being a 2×2 lower triangular matrix

$$\mathbf{H}_n = \begin{bmatrix} h_{SD} & 0 \\ \alpha_n b_n h_{RD_n} h_{SR_n} & \alpha_n h_{SD} \end{bmatrix},$$

and $\alpha_n = \frac{1}{\sqrt{1 + b_n^2 |h_{RD_n}|^2}}$ is the noise whitening factor.

B. Channel State Information and Amplification Coefficients

For convenience, let

$$\mathbf{h} = [h_{SD}, h_{SR_1}, h_{SR_2}, ..., h_{SR_N}, h_{RD_1}, h_{RD_2}, ..., h_{RD_N}]^T,$$

be the set of channel gains from S to D, S to R_n, and R_n to D. As in [6]–[8], it is assumed that $\mathbf{h}$ remains constant during at least one cooperative frame and D has perfect knowledge of the channel gains in all links. In addition, as recently considered in [12], [13], such complete channel state information (CSI) can also be made available at each relay, but not the source, thanks to a feedback channel from the destination to the relays. This assumption is practically reasonable, since a relay can be indeed controlled directly by the service provider.

Let $q_{n1} = \mathbb{E}[|x_{n,1}|^2]$ and $q_{n2} = \mathbb{E}[|x_{n,2}|^2]$, where $\mathbb{E}[.]$ denotes the expectation operator. It means that S spends an average power of $q_{n1}E_S$ and $q_{n2}E_S$ in the corresponding broadcasting phase and cooperative phase, respectively, where q_{n1} and q_{n2} are strictly positive. Furthermore, let $\mu_n E_S$ be the *instantaneous* power allocated to the relay R_n in one cooperative frame. With complete CSI, the relays can adapt their instantaneous transmit powers to the current channel conditions. Thus, the instantaneous value of μ_n depends on $\mathbf{h}$, i.e., $\mu_n = \mu_n(\mathbf{h})$. Such a variation is reflected in the use of an instantaneous power amplification coefficient at each relay R_n, which can be written as

$$b_n = \sqrt{\frac{\mu_n(\mathbf{h})\rho}{|h_{SR_n}|^2 q_{n1}\rho + 1}}, \tag{2}$$

where $\rho = E_S/N_0$ is the normalized SNR. With the TAPC, it is assumed that the total average power available for all relays is limited to P, i.e.,

$$\sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P, \tag{3}$$

with $f(\mathbf{h})$ be the probability density function of the channel state $\mathbf{h}$. Under a more realistic scenario, each relay can also have its own average power constraint limited to a fixed P_n due to the constraint on each individual RF chain. Such a constraint is referred to as individual average power constraint (IAPC) and can be expressed as

$$\int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P_n. \tag{4}$$

It is also assumed that $\sum_{n=1}^{N} P_n \geq P$. Otherwise, the TAPC is always satisfied and one needs to consider only the IAPC. Finally, it is worth mentioning that when the relays only have knowledge of the channel distribution information (CDI) of the S-R link, the amplification coefficient at relay R_n is given as [2], [8], [10]:

$$b_n^{(CDI)} = \sqrt{\frac{P\rho}{N\left(\mathbb{E}[|h_{SR_n}|^2] q_{n1}\rho + 1\right)}}, \tag{5}$$

On the other hand, when the CSI of the S-R link is available at the relays, each relay R_n can use the channel inversion with the following amplification coefficient [3], [6]:

$$b_n^{(CI)} = \sqrt{\frac{P\rho}{N\left(|h_{SR_n}|^2 q_{n1}\rho + 1\right)}}. \tag{6}$$

III. Optimal Power Adaptation Scheme

In this section, we shall develop the optimal power sharing schemes among the relays to maximize the average MI with Gaussian inputs. We first consider the TAPC, and then both the TAPC and IAPC simultaneously. As shall be shown shortly, the optimization problems for the considered NAF system are not convex. As an alternative, we propose a novel method by relaxing the original problem and show that the solution of the modified problem is still globally optimal. Finally, we provide some important insights regarding the proposed schemes.

When the source S has no CSI knowledge and uses Gaussian inputs, it has been shown in [8] that the optimal input covariance matrix will be the diagonal matrix given as

$$\mathbf{Q} = diag(q_{11}, q_{12}, q_{21}, q_{22}, ..., q_{N1}, q_{N2}).$$

From (1), the instantaneous conditional MI between the input and output given CSI at the destination can then be expressed as

$$I(\mathbf{x},\mathbf{y}|\mathbf{h}) = \log\det[\mathbf{I}_{2N} + \rho\mathbf{H}^{\dagger}\mathbf{H}\mathbf{Q}], \quad (7)$$

where † denotes the Hermitian operator, $log(.)$ is the base-2 logarithm. For the sake of convenience, let $x_n = |h_{RD_n}|^2$, $y = |h_{SD}|^2$, $z_n = |h_{SR_n}|^2$. After some manipulations, one has

$$I(\mathbf{x},\mathbf{y}|\mathbf{h}) = \sum_{n=1}^{N} \log\left(\Gamma_n[\mu_n(\mathbf{h})]\right) = \sum_{n=1}^{N} \log\left(\frac{\Gamma_{n1}[\mu_n(\mathbf{h})]}{\Gamma_{n2}[\mu_n(\mathbf{h})]}\right), \quad (8)$$

where

$$\Gamma_{n2}[\mu_n(\mathbf{h})] = \mu_n(\mathbf{h})x_n + (\rho q_{n1} z_n + 1), \quad (9)$$

and

$$\begin{aligned}\Gamma_{n1}[\mu_n(\mathbf{h})] &= \mu_n(\mathbf{h})\left(\rho q_{n1} y + \rho q_{n1} z_n + 1\right)\rho x_n \\ &+ \left(\rho^3 q_{n1}^2 q_{n2} z_n y^2 + \rho^2 q_{n1} q_{n2} y^2 + \rho^2 q_{n1}^2 z_n y\right. \\ &\left.+ \rho^2 q_{n1} q_{n2} z_n y + \rho q_{n1} z_n + \rho q_{n1} y + \rho q_{n2} y + 1\right). \quad (10)\end{aligned}$$

A. Total average power constraint

In this subsection, we consider only the TAPC in (3) imposed on all N relays. The set of optimal instantaneous power allocations at the relays $\{\mu_n^*(\mathbf{h})\}$ to maximize the above MI can be obtained by solving the following optimization problem

$$\max_{\substack{\mu_n(\mathbf{h})\geq 0\\ n=\overline{1,N}}} \int_{\mathbf{h}} I(\mathbf{x},\mathbf{y}|\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \quad s.t \ \sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P. \quad (11)$$

Let $I(\mathbf{x},\mathbf{y}|\mathbf{h}) = I(\boldsymbol{\mu}(\mathbf{h}))$ where $\boldsymbol{\mu}(\mathbf{h}) = [\mu_1(\mathbf{h}),\ldots,\mu_{\mathbf{N}}(\mathbf{h})]$. Differentiating the objective function in (11) with respect to $\mu_n(\mathbf{h})$ we have

$$\begin{aligned}&\frac{\partial}{\partial\mu_n(\mathbf{h})}\int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} = \frac{-A_n}{\ln(2)[C_n\mu_n^2(\mathbf{h}) + D_n\mu_n(\mathbf{h}) + E_n]} f(\mathbf{h}),\\ &\frac{\partial^2}{\partial\mu_n^2(\mathbf{h})}\int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} = \frac{A_n[2C_n\mu_n(\mathbf{h}) + D_n]}{\ln(2)[C_n\mu_n^2(\mathbf{h}) + D_n\mu_n(\mathbf{h}) + E_n]^2} f(\mathbf{h}),\\ &\frac{\partial^2}{\partial\mu_m(\mathbf{h})\partial\mu_n(\mathbf{h})}\int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} = 0,\ (m \neq n),\end{aligned} \quad (12)$$

where

$$\begin{aligned}A_n &= \rho^2 x_n(\rho q_{n1} z_n + 1)(\rho q_{n1} q_{n2} y^2 + q_{n2} y - q_{n1} z_n),\\ C_n &= \rho^2 x_n^2(\rho q_{n1} y + \rho q_{n1} z_n + 1),\\ D_n &= \rho x_n(\rho q_{n1} z_n + 1)(2\rho q_{n1} y + \rho q_{n2} y\\ &+ \rho q_{n1} z_n + \rho^2 q_{n1} q_{n2} y^2 + 2),\\ E_n &= (\rho q_{n1} y + 1)(\rho q_{n2} y + 1)(\rho q_{n1} z_n + 1)^2.\end{aligned} \quad (13)$$

It can be verified from (12) and (13) that the optimization problem in (11) is not concave. As a consequence, KKT conditions cannot be applied directly. To overcome this drawback, let first define the feasible set for the problem as

$$\mathcal{F} = \{\boldsymbol{\mu}(\mathbf{h}) \in \mathbb{R}^N | \mu_n(\mathbf{h}) \geq 0, \sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P\}.$$

Let $\gamma_n = \rho q_{n1} q_{n2} y^2 + q_{n2} y - q_{n1} z_n$. For each relay R_n, divide the domain of $\mathbf{h}$ into two disjoint regions, $\mathbf{h}_{\mathbf{P}}^{(n)} = \{\mathbf{h}|\gamma_n \geq 0\}$ and $\mathbf{h}_{\mathbf{N}}^{(n)} = \{\mathbf{h}|\gamma_n < 0\}$. In the domain $\mathbf{h}_{\mathbf{P}}^{(n)}$ we have $A_n \geq 0$, thus $I(\boldsymbol{\mu}(\mathbf{h}))$ is a decreasing function of $\mu_n(\mathbf{h})$ in this domain. Let $\boldsymbol{\mu}^{(n)}(\mathbf{h})$ be a vector obtained by replacing the element $\mu_n(\mathbf{h})$ in $\boldsymbol{\mu}(\mathbf{h})$ by zero. It is straightforward to see that

$$I(\boldsymbol{\mu}(\mathbf{h})) \leq I(\boldsymbol{\mu}^{(n)}(\mathbf{h})),\ \ \forall \mathbf{h} \in \mathbf{h}_{\mathbf{P}}^{(n)}.$$

and for any $\boldsymbol{\mu} \in \mathcal{F}$ and $n = 1, 2, ..., N$ we have

$$\begin{aligned}&\int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} = \int_{\mathbf{h}_{\mathbf{N}}^{(n)}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h}\\ &+ \int_{\mathbf{h}_{\mathbf{P}}^{(n)}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h}\\ &\leq \int_{\mathbf{h}_{\mathbf{N}}^{(n)}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} + \int_{\mathbf{h}_{\mathbf{P}}^{(n)}} I\left(\boldsymbol{\mu}^{(n)}(\mathbf{h})\right) f(\mathbf{h})\, d\mathbf{h}.\end{aligned} \quad (14)$$

Hence, the objective function in (11) can then be upper-bounded as

$$\begin{aligned}&\int_{\mathbf{h}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} \leq\\ &\frac{1}{N}\sum_{n=1}^{N}\left(\int_{\mathbf{h}_{\mathbf{N}}^{(n)}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h} + \int_{\mathbf{h}_{\mathbf{P}}^{(n)}} I\left(\boldsymbol{\mu}^{(n)}(\mathbf{h})\right) f(\mathbf{h})\, d\mathbf{h}\right)\end{aligned} \quad (15)$$

and one achieves the equality when $\mu_n(\mathbf{h}) = 0 \forall \mathbf{h} \in \mathbf{h}_{\mathbf{P}}^{(n)}, n = 1, 2, ..., N$. Using (15), we then have the following modified optimization problem:

$$\begin{aligned}&\max_{\substack{\mu_n(\mathbf{h})\geq 0\\ n=\overline{1,N}}} \frac{1}{N}\sum_{n=1}^{N}\left(\int_{\mathbf{h}_{\mathbf{N}}^{(n)}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h}\right.\\ &\left.+ \int_{\mathbf{h}_{\mathbf{P}}^{(n)}} I\left(\boldsymbol{\mu}^{(n)}(\mathbf{h})\right) f(\mathbf{h})\, d\mathbf{h}\right)\\ &s.t.\ \ \sum_{n=1}^{N}\int_{\mathbf{h}_{\mathbf{N}}^{(n)}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P.\end{aligned} \quad (16)$$

We can now verify that the Hessian matrix of the objective function in (16) is diagonal with strictly negative elements. Therefore, (16) is now a concave optimization problem and any solution in the form of $\mu_n(\mathbf{h}) = \mu_{\mathbf{n}}^*(\mathbf{h})$ is *globally optimal*. The problem in (16) can then be solved using the KKT conditions. The globally optimal instantaneous power allocation $\mu_n^*(\mathbf{h})$ for the NAF system with the TAPC is given as

$$\mu_n^*(\mathbf{h}) = \begin{cases} 0, & A_n \geq 0\\ \left(\frac{-D_n + \sqrt{D_n^2 - 4C_n\left(E_n + \frac{A_n}{\ln(2)N\lambda^*}\right)}}{2C_n}\right)^+, & A_n < 0,\end{cases} \quad (17)$$

where $\lambda^* > 0$ is a unique constant that satisfies $\sum_{n=1}^{N}\int_{\mathbf{h}} \mu_n^*(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} = P$ and can be easily found numerically.

B. Individual average power constraint

Now we consider the NAF system under both the TAPC and IAPC. Similar to previous subsection, by dividing the channel vectors in two different subsets we have the optimization problem for the optimal power allocation $\mu_n(\mathbf{h})$ at each relay R_n as

$$\begin{aligned}&\max_{\substack{\mu_n(\mathbf{h})\geq 0\\ n=\overline{1,N}}} \frac{1}{N}\sum_{n=1}^{N}\left(\int_{\mathbf{h}_{\mathbf{N}}^{(n)}} I(\boldsymbol{\mu}(\mathbf{h})) f(\mathbf{h})\, d\mathbf{h}\right.\\ &\left.+ \int_{\mathbf{h}_{\mathbf{P}}^{(n)}} I\left(\boldsymbol{\mu}^{(n)}(\mathbf{h})\right) f(\mathbf{h})\, d\mathbf{h}\right)\\ &s.t.\ \begin{cases}\sum_{n=1}^{N}\int_{\mathbf{h}_{\mathbf{N}}^{(n)}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P\\ \int_{\mathbf{h}_{\mathbf{N}}^{(n)}} \mu_n(\mathbf{h}) f(\mathbf{h})\, d\mathbf{h} \leq P_n.\end{cases}\end{aligned} \quad (18)$$

It can be seen that this is also a concave optimization problem and can be solved by using the KKT conditions. The globally optimal instantaneous power allocation $\mu_n^*(\mathbf{h})$ for the NAF protocol with both the TAPC and IAPC is finally expressed as

$$\mu_n^*(\mathbf{h}) = \begin{cases} 0, & A_n \geq 0\\ \left(\frac{-D_n + \sqrt{D_n^2 - 4C_n\left(E_n + \frac{A_n}{\ln(2)N(\lambda^* + \lambda_n^*)}\right)}}{2C_n}\right)^+, & A_n < 0\end{cases} \quad (19)$$

where $\lambda^* > 0$ and $\lambda_n^* \geq$ are constant satisfying

$$\begin{cases} \sum_{n=1}^{N} \int_{\mathbf{h}} \mu_n^*(\mathbf{h}) f(\mathbf{h}) d\mathbf{h} = P, \int_{\mathbf{h}} \mu_n^*(\mathbf{h}) f(\mathbf{h}) d\mathbf{h} \leq P_n, \\ \lambda_n^* \left(P_n - \int_{\mathbf{h}} \mu_n^*(\mathbf{h}) f(\mathbf{h}) d\mathbf{h} \right) = 0. \end{cases} \quad (20)$$

The two parameters λ^* and λ_n^* depend on the channel distribution $f(\mathbf{h})$, the SNR ρ, and the average power allocations P and $\{P_n\}$, and they can be found numerically.

C. Discussions on the Solutions

In this subsection, we shall discuss the meaning of the optimal solutions. The main focus will be on the power allocated to each relay and the probability of relays being active. For simplicity, we consider the system with two relays. We also assume that the source allocates the same power for all phases, i.e., $q_{11} = q_{12} = q_{21} = q_{22}$. In the case of using the IAPC, it is also assumed that the IAPCs on the relays are the same and equal to $P_n = \eta P$ where $\eta \geq \frac{1}{N} P$.

It can be seen that the solution in (17) is a water-filling based scheme. Moreover, when we consider the IAPCs, the obtained solution in (19) is the extended water-filling scheme when the vessels have both bottoms and lids. It can also be observed that the average power allocated to a particular relay not only depends on its related links but also depends on all the other links involved in the system. Let $Pr_{n,off} = \Pr[\mu_n(\mathbf{h}) = \mathbf{0}]$ be the probability that a particular relay R_n is silent at a given frame. The probability of that relay being active at the same frame is therefore $Pr_{n,on} = \Pr[\mu_n(\mathbf{h}) > 0] = 1 - Pr_{n,off}$.

Consider the case with only TAPC, it can be seen from (17) that the relay R_n will be turn-off in either two scenarios: i) $A_n \geq 0$; or ii) $A_n < 0$ and $(E_n \ln(2) N \lambda^* + A_n) \geq 0$. Notice from the first condition and (13) that the relay R_n shall keep silent whenever $(\rho q_{n1} q_{n2} y^2 + q_{n2} y - q_{n1} z_n) \geq 0$. Hence, $Pr_{n,on}$ is small at high SNRs. Furthermore, when power is poured equally into two cooperative phases ($q_{n1} = q_{n2}$), the relay R_n also remains off whenever $y > z_n$, i.e., the S-D link is stronger than the S-R_n link. On the other hand, the second condition $A_n < 0$ indicates the relay R_n should not be used if

$$\frac{\rho^2 x_n (q_{n1} z_n - \rho q_{n1} q_{n2} y^2 - q_{n2} y)}{(\rho q_{n1} y + 1)(\rho q_{n2} y + 1)(\rho q_{n1} z_n + 1)} < \lambda^* \ln(2) N. \quad (21)$$

It is obvious from (21) that the relay R_n remains silent if x_n is small, i.e., one has a weak R_n-D channel. It can also be seen from the above equation that the relays are rarely used at low SNRs. If the system is symmetric, the probabilities of using the relays are all the same and the allocated powers to all the relays are equal. In the asymmetric scenario, without loss of generality, assume that relay R_2 has a stronger overall link over relay R_1, i.e., $\mathbb{E}[x_2] > \mathbb{E}[x_1]$ and $\mathbb{E}[z_2] > \mathbb{E}[z_1]$. Then it is straightforward to show that

$$\begin{aligned} & \mathbb{E}_{\mathbf{h}} \left[\frac{\rho^2 x_2 \left(q_{21} z_2 - \rho q_{21} q_{22} y^2 - q_{22} y \right)}{(\rho q_{21} y + 1)(\rho q_{22} y + 1)(\rho q_{21} z_2 + 1)} \right] \\ > & \mathbb{E}_{\mathbf{h}} \left[\frac{\rho^2 x_1 \left(q_{11} z_1 - \rho q_{11} q_{12} y^2 - q_{12} y \right)}{(\rho q_{11} y + 1)(\rho q_{12} y + 1)(\rho q_{11} z_2 + 1)} \right] \end{aligned} \quad (22)$$

From (21) and (22), it is clear that that $P_{2,off} < P_{1,off}$. As such, relay R_2 is allocated more average power over relay R_1. When the IAPC is considered, it is also straightforward to show that a relay with a stronger overall channel is allocated at most ηP. The probability of using such a relay is smaller or equal to that in the case of having only the TAPC. To illustrate the probabilities of using the relays, Fig. 2 shows $\{Pr_{on}\}$ against SNR ρ in different configurations for the NAF protocol using the power adaptation schemes in (17) and (19). For simplicity, uniform power allocation is assumed at the source, i.e., $q_{11} = q_{12} = q_{21} = q_{22} = 0.5$, and $P = 2$. For the channels, we adopt the distance-dependent path-loss model [15] such that the channel variances depend on the path-loss exponent, i.e., $\mathbb{E}[|h_{SD}|^2] = \phi_{SD} = d_{SD}^{-v}, \mathbb{E}[|h_{SR_n}|^2] = \phi_{SR_n} = d_{SR_n}^{-v}, \mathbb{E}[|h_{RD_n}|^2] = \phi_{RD_n} = d_{RD_n}^{-v}$, where $v = 3$ is the path-loss exponent, and $d_{SD}, d_{SR_n}, d_{RD_n}$ are the distances between S-D, S-R_n, R_n-D, respectively. We consider the following three scenarios: i) Symmetric system (Sym) in which $d_{SD} = d_{SR_1} = d_{SR_2} = d_{RD_1} = d_{RD_2} = 1$; ii) Asymmetric system with only the TAPC (Asym1); iii) Asymmetric system with both the TAPC and IAPC (Asym2) in which $\eta = 0.8$. In both Asym1 and Asym2, the channel distances are set as $d_{SD} = 1, d_{SR_1} = d_{RD_1} = 2, d_{SR_2} = d_{RD_2} = 0.5$. Observe from Fig. 2 that the relays are not likely to be used in very low and very high SNR regimes. In addition, the probabilities of using the two relays are identical in the symmetric system. This behaviour does not depend on the presence of the IAPC as long as $\eta \geq 0.5$. In the asymmetric scenarios, the relay having stronger S-R-D link is used more often. In Asym2, the presence of the IAPC decreases the probability of using the relay with stronger overall channels and increases that of the other relay.

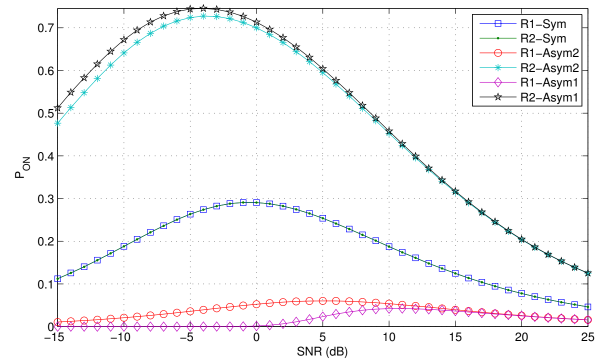

Fig. 2. The probabilities P_{on} of the NAF protocol.

IV. Illustrative Results

In this section, simulation results are presented to quantify the gain provided by the system having full CSI at the relays over the two conventional systems using CDI and CI. For brevity of the presentation, we only consider the systems having two relays and it is assumed that the source allocates power equally in each phase with $P = 2$. Unless otherwise stated, the distance-dependent path-loss model [15] is adopted. We shall evaluate four different power adaptation schemes: i) CSI-1 - a system with full CSI under the TAPC; ii) CSI-2 - a system with full CSI under both the TAPC and IAPC; iii) CDI - a system using channel distribution information; and iv) CI - a system using channel inversion.

Fig. 3 first plots the MI of the CSI-1, CDI, and CI systems in a symmetric configuration. Note that all the considered system have the same TAPC. It can be seen from Fig. 3 that the CSI-1 system significantly outperforms the CDI and CI systems with the gains of around 0.8dB and 1dB, respectively.

The impressive gains offered by CSI can also be observed in the asymmetric configuration, which is a more realistic model.

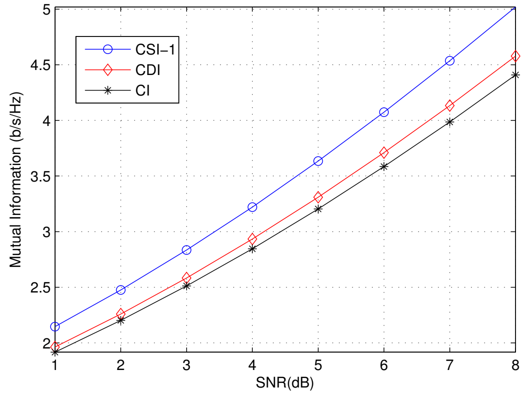

Fig. 3. The MI of different NAF systems over a symmetric channel with $d_{SD} = d_{SR1} = d_{SR2} = d_{RD1} = d_{RD2} = 1$.

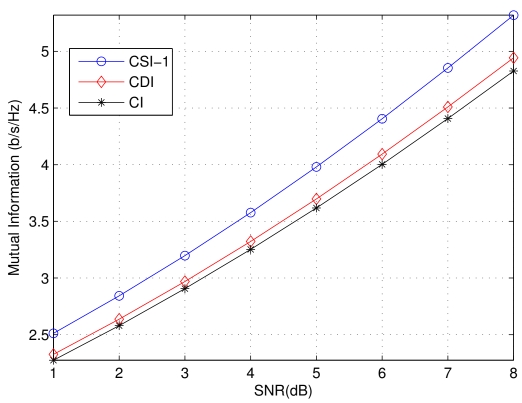

Fig. 4. The MI of different NAF systems over an asymmetric channel with $d_{SD} = 1, d_{SR1} = d_{RD1} = 1.5, d_{SR2} = d_{RD2} = 0.5$.

In particular, Fig. 4 provides the MI of the CSI-1, CDI, and CI systems in an asymmetric configuration with $d_{SD} = 1, d_{SR1} = 1.5, d_{RD1} = 1.5, d_{SR2} = 0.5, d_{RD2} = 0.5$. Observe from Fig. 4 that the gains are as much as 0.75dB and 1dB over the other two CDI and CI systems, respectively. Again, these gains come from the flexibility in sharing the total power constraint between two relays, thanks to the knowledge of CSI. Note that we also observe similar gains in other asymmetric setups.

Finally, Fig. 5 compares the MI of the two NAF systems with full CSI under only the TAPC and under both the TAPC and IAPC over the above asymmetric channel, i.e., with $\eta = 0.5, d_{SD} = 1, d_{SR1} = d_{RD1} = 1.5, d_{SR2} = d_{RD2} = 0.5$. As expected, there is a small penalty by imposing individual power constraint on each relay. The loss is of about 0.1dB at various SNR regions. However, the system with CSI under both total and individual power constraints still outperforms the two CDI and CI systems.

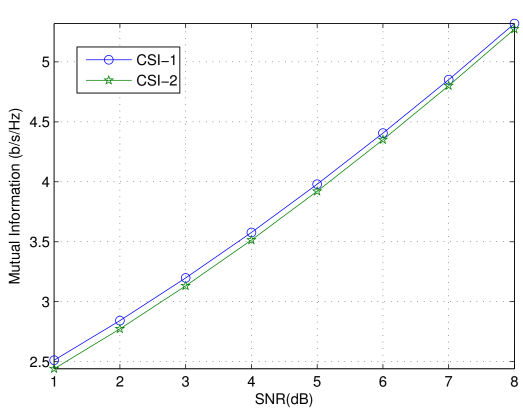

Fig. 5. The MI of the two NAF systems with full CSI under the TAPC and under both the TAPC and IAPC over an asymmetric channel with $\eta = 0.5, d_{SD} = 1, d_{SR1} = d_{RD1} = 1.5, d_{SR2} = d_{RD2} = 0.5$.

V. Conclusions

This paper developed optimal power sharing schemes among multiple relay nodes for a non-orthogonal amplify-and-forward (NAF) half-duplex relay network where multiple relays have full CSI. The closed-form optimal solutions were derived in various conditions. Specifically, under only the total average power constraint, extended water-filling solutions over time and space were proposed. When one has both the total average and individual average power constraints, the optimal solutions are the extended water-filling schemes with the vessels having both bottoms and lids. Numerical results showed that having CSI at the relays provides higher MI compared to the systems having only channel distribution information (CDI) and channel inversion (CI) at the relays.

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