Indicative Conditionals

Anthony S. Gillies
Rutgers University

1 The Target

Sometimes philosophy gets started by pointing. Setting out the target for theories of *indicative conditionals* is a case in point. So some examples to point at:

(1) a. If the gardener didn't do it, (then) it was the butler.
b. If your blue marble is in the box, then your red one might be under the couch.

Our target is the class of ordinary *if ... then ...* constructions like these. They express conditional information: information about what is or might or must be, if such-and-such is or turns out to be the case.

Indicatives (the name is not great but is entrenched) stand apart from two other sorts of conditional constructions. First, from subjunctive or counterfactual conditionals. An Adams-pair:

(2) a. If Oswald didn't kill Kennedy, then someone else did.
b. If Oswald hadn't killed Kennedy, someone else would have.

The conditional information these traffic in is different: (2a) is an indicative and true; it's counterfactual cousin (2b) is false. Subjunctive conditionals — that is, those with a distinctive tense/aspect marking like (2b) — say something about would or might *have been* if such-and-such *had been*.

(The "subjunctive" marking doesn't go exactly with counterfactuality. Indeed, it's neither necessary (as Anderson (1951)-type examples show) nor sufficient (as sportscasterese-type conditionals show). See von Fintel 1998 (and the references therein) for the status of the connection between "subjunctive" marking and counterfactuality.)

Second, indicatives stand apart from *biscuit conditionals*. Austin's (1956) example:

(3) There are biscuits on the sideboard if you want them.

A fine thing to say, but not normally a thing that expresses conditional information. While it is open to connect a story about indicatives and counterfactuals, and while it is open to connect a story about indicatives and biscuit conditionals, we will be setting such outreach aspirations aside. Having pointed a few times, let's get going.

2 Landscape

Saying that indicatives carry conditional information isn't yet to say very much. We want a better grip on:

i. Just what that conditional information is. This is a story about what indicatives in natural language mean (what semantic values they have) and how they are used in well-run conversation (what their pragmatic profile looks like).

ii. Just how sentences of natural language express those meanings. This is a story about how *if* interacts with the rest of our language (that holds good both for compositionally deriving how *if*s express conditional information and for seeing how conditional sentences contribute their conditional meanings to embedding environments).

Suppose that semantic values (whatever they are) determine truth values at points of evaluation (worlds, situations, whatever). This holds, let's assume, for both iffy and non-iffy sentences. Not wholly innocent — as we'll see — but a place to start.

Let's distinguish between (i) conditional sentences of natural language and (ii) conditional connectives of some formal language that serves to represent the logical forms of conditional sentences. The aim is then to associate an *if*-in-English (via well-behaved mapping that we won't bother with) with an *if*-in-the-formal-language that then is associated (via a well-behaved mapping we will bother with) with its semantic value. (There are ways to express conditional information in natural language without resorting to *if*s (in some languages, it's the only way). The issues in this particular ballpark won't matter too much for our purposes so we can (pretty) safely focus on the *if . . . then* of English.) For simplicity we'll stick with a simple propositional language (with sentential connectives $\neg, \wedge, \vee, \supset$) together with a connective $(if\ \cdot)(\cdot)$ for the indicative. (Whether indicatives can be represented by a binary conditional connective in a regimented intermediate language is also, as we'll see, up for grabs.) This indirect route to assigning meanings to indicatives isn't required, is in-principle dispensable (assuming our mappings are well-behaved), but makes for an unobstructed view.

One way to make sense out of the idea of conditional information is to let some version of the Ramsey Test guide us. Here's what Ramsey (1929/1990: 155) says: "If two people are arguing 'If p, will q?' and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q ...." The adding this involves can't be feigned or hypothetical belief. (Thomason is credited with this observation in van Fraassen 1980.) If it were, the Ramsey Test would go wrong:

(4) If my students are cheating in class, then I will not discover it (because they're so clever).

The state I get into by feigning belief in the antecedent won't be one in which I believe the consequent. The right way to understand augmenting for the Ramsey test is restricting a body of information by the information carried by the antecedent. A more neutral version of the Ramsey Test skirts this: the conditional information carried by $(\textit{if}\ P)(Q)$ is true/accurate/acceptable in a situation iff Q is true/accurate/acceptable in that situation-plus-the-information-that-P.

Theories of indicatives are constrained by the patterns of intuitive entailment they participate in. So what-intuitively-entails-what is important. We want to explain those patterns as best we can. Consider whether indicatives generally go in for a *deduction theorem* (a.k.a. *conditional proof*):

(5) $\Gamma \cup \{P\} \vDash Q$ iff $\Gamma \vDash (\textit{if}\ P)(Q)$

If so, then we have an especially tight connection between what $(\textit{if}\ \cdot)(\cdot)$ means and what we say about entailment. The point is that 'entailment' is as much part of the theoretical machinery as is anything. Take it as given that what we may say about *if*s can interact in non-trivial ways with what we say about entailment. Then the best route to the best theory may be one in which what we say about entailment bends to the will of the *if*s as much as the other way around.

A concrete example: what Stalnaker (1975) calls the *direct argument*.

(6) a. Either the butler did it or the gardener did it.
$P \vee Q$
b. So: if the butler didn't do it, then the gardener did.

$(\mathit{if}\ \neg P)(Q)$

Seems like (6a) entails (6b). The trouble is that treating such *or*-to-*if* arguments as entailments seems to get us quickly to the conclusion that indicatives mean just what their corresponding material conditionals do. Suppose semantic values are something that determine truth values and that entailment is preservation of truth. And assume the direct argument is an entailment (that $P \vee Q \vDash (\mathit{if}\ \neg P)(Q)$). Then $P \supset Q$ entails the indicative $(\mathit{if}\ P)(Q)$.

Argument. Suppose $P \supset Q$ is true (at a world w). Then so is $\neg P \vee Q$. Thus by the direct argument so is $(\mathit{if}\ P)(Q)$ (at w). (The entailment in the other direction isn't usually disputed.) □

We can follow where the argument leads or we can look for places to get off the boat.

Another example: a *boundedness* argument (Gibbard 1981; Veltman 1985). Indicatives seem to fall somewhere on the spectrum of logical strength between strict implication and material implication: when P entails Q the indicative $(\mathit{if}\ P)(Q)$ must be true and the falsity of $P \supset Q$ entails the falsity of the indicative $(\mathit{if}\ P)(Q)$. That last bit — that indicatives are bounded from below by material conditionals — is equivalent to saying that they go in for modus ponens. And then there is *import/export*:

(7) a. If the gardener is away and the driver is away, then the mansion is empty.
$(\mathit{if}\ (P \wedge Q))(R)$

b. If the gardener is away, then if the driver is away then the mansion is empty.
$(\mathit{if}\ P)((\mathit{if}\ Q)(R))$

Pairs like these seem to be mutual entailers. Again it seems that indicative conditionals and material conditionals say the same thing, that the truth of $P \supset Q$ is sufficient for the truth of the indicative $(\mathit{if}\ P)(Q)$.

Argument. Suppose $\neg P$ is true. Since $(\neg P \wedge P)$ entails Q and indicatives are true when their antecedents entail their consequents, we have that

(8) $(\mathit{if}\ (\neg P \wedge P))(Q)$

is true. By import/export

(9) $(\textit{if } \neg P)((\textit{if } P)(Q))$

is true. And since $\neg P$ is true and material conditionals are the lower bound, the material conditional $(\textit{if } P)(Q)$ is true, too. Now suppose Q is true. The argument goes just as before, except now we rely on the fact that $Q \wedge P$ entails Q. So if $\neg P \vee Q$ is true so is $(\textit{if } P)(Q)$. □

As before, we either follow the argument to the material conditional or we look for escape routes. That is a tidy way of organizing our choices.

One escape route — flat-footedly denying modus ponens — isn't very popular. Though, as things go in philosophy, that's not because no one has pushed for it. McGee (1985) sees boundedness arguments as pitting modus ponens against import/export. He thinks there are counterexamples to modus ponens and not to import/export, so that forces his choice. (The counterexamples all involve a right-nested indicative we think true (and whose antecedent we think true) even though we do not especially think the embedded conditional on its own is true. There is a not-small literature on the status of the counterexamples.) Others deny some mixture of modus ponens and import/export, but not obviously for reasons connected to boundedness-style arguments (Lycan (2001) denies both import/export and (for good measure) modus ponens, but not without cost).

3 Horseshoe

The *horseshoe theory* is both the simplest response to arguments like the boundedness argument and the simplest story about indicatives: $(\textit{if } \cdot)(\cdot)$ has just the truth conditions that $\supset$ does.

Horseshoe Theory Indicatives are material conditionals:

$[\![(\textit{if } P)(Q)]\!]^{c,w} = 1$ iff either P is false or Q is true (at w, in c).

This obviously treats indicatives as truth-functional.

Assuming truth-functionality, is there any other way of assigning truth conditions? No.

Argument. This

(10) If Jimbo is taller than 6 feet, then Jimbo is taller than 5 feet.
$(\textit{if } P)(Q)$

should be true (at w) no matter what. Possible heights for Jimbo: (i) over 6 feet (antecedent true), (ii) between 5 and 6 feet (antecedent false, consequent true), and (iii) less than 5 feet (antecedent and consequent false). These are the conditions under which $P \supset Q$ is true. So there's one indicative true when it's corresponding horseshoe is. Assuming truth-functionality, this has to be true for *any* indicative. Moreover, indicatives must be false when their corresponding horseshoes are false: otherwise, all indicatives would always be true. So the horseshoe is the only choice. □

The horseshoe theory is, gently put, not the most widely held view these days. But it's not as though it has nothing going for it. Indicatives certainly behave like material conditionals in mathematical contexts. And the horseshoe theory says that instances of the direct argument, import/export, and modus ponens — an impressive who's who in properties conditionals seem to have — strike us as entailments because they are.

It's easy for the horseshoe theory to claim entailments (to conditionals). That is because the material conditional is so weak. The problem is — for just that reason — we have *more* entailments (to conditionals) than we want. (In)famously among them: the paradoxes of material implication.

(11) a. Carl came alone.
??So: if Carl came with Lenny, neither came.
b. Billy got here first.
??So: if Alex got here before Billy, Billy got here first.

These don't seem like entailments even though the truth of either $\neg P$ or Q at w secures the truth of $P \supset Q$ at w. The horseshoe champion owes us some answers.

What we need is an extra-semantic explanation saying that these are entailments, all right, but ones we can live with because the pragmatic facts about conditionals — how they are reasonably and appropriately used — explains their weirdness. That is Grice's (1975) strategy. Conditionals like the one in (11a) are true but ruled out on general grounds. Once we know Carl came alone, it is pointless (though true) to say that either he didn't come with Lenny or neither came. Since it is pointless, as Lewis (1976: p.142) says, "also it is worse than pointless: it is misleading." Ditto for (11b): if you are in a position to say Q then you shouldn't be in the business of saying the weaker conditionalized thing. This kind of explaining away is principled. It doesn't rely on anything special about conditionals: once we take on board the

Gricean picture of conversational implicature, we can say something about why the examples in (11) are odd even though the underlying entailments are real. (Lewis (1976) initially goes for this defense of the horseshoe theory for indicatives, using (part of) it to explain why assertability of indicatives goes with their conditional probability and not with the probability of their truth.)

Another kind of explanation: say that indicatives *conventionally* implicate something extra. Jackson (1991) develops this defense, saying that they require "robustness": when you issue an indicative $(if\ P)(Q)$ it conventionally implicates that your credence in $P \supset Q$ conditional on P — which is to say that the probability of Q conditional on P — is high enough. (Lewis eventually drops the conversational story and instead goes for a variant of Jackson's conventional implicature story.) So *if* is supposed to be like *but*: truth-functional in meaning with an extra, conventionally encoded signal. The problem with, for instance, (11a) is that our reason for thinking the conditional is true is that we think the antecedent false — that's a paradigm case where robustness fails (we drop the conditional upon learning that — to our surprise — Carl came with Lenny). Similarly for (11b). There is little independent evidence of this conventional implicature however (see Bennett 2003: §16 and Edgington 2009: §4.2).

The implicature explanations differ (in mechanism and commitment) but have a lot in common. Both say that it is some extra-semantic fact about using indicatives that explains away the oddness of the entailments in (11). Both say that when it comes to indicatives it's assertability preservation (not some notion of entailment tied to the semantic values of indicatives) that matters and that judgments about entailment may well be clouded by judgments about assertability preservation.

But both also have difficulty explaining facts about indicatives occurring unasserted in embedded environments. The mechanisms for the assert-the-stronger conversational implicature explanation don't apply. And we have no story about how a special-purpose robustness conventional implicature projects out from under embedding constructions. That is too bad because conditionals *do* occur in larger environments and the horseshoe theory inherits some pretty bad commitments when they do. Everyone knows it was either the gardener, the driver, or the butler. So when the novice on the beat declares that if the gardener's alibi checks out then you can arrest the butler, you rightly object:

(12) a. It's not so that if the gardener didn't do it then the butler did.

b. Just because the gardener didn't do it that doesn't mean the butler did.

In (12a) we have an ordinary indicative under a wide scope sentence negation. (This may be inelegant English, but it's still English.) What you say doesn't entail that the gardener didn't do it. (And it doesn't entail that the butler didn't do it.) This line of reasoning is not good:

(13) Since the driver is still a suspect, it's not so that if the gardener didn't do it then the butler did.
??So: it wasn't the gardener.

The point is that you can be all signed-up for $\neg\,(\mathit{if}\;\; P)(Q)$ without being signed-up for the truth of P. But if $\neg(P \supset Q)$ is true then so is P. This is the price material conditionals pay for being so weak: their negations are strong. So we need to explain away this entailment. But the implicature gymnastics invoked thus far don't seem up to it. Maybe some *other* pragmatic mechanism is at work — perhaps the apparent widescope negation isn't a regular negation at all but is really a denial or metalinguistic negation operator (Horn 2001): when you say (13a) you are not asserting the negation of a conditional but you are denying that you will assert the embedded conditional. (This is Grice's reply. Horn seems happy enough to follow him in part because the negation is so awkward — a test, he says, for metalinguistic negation. Merely being resistant to embedding under negation lumps things wrong, though: some modals don't like it (*may* and *must*) even though others don't mind at all (*can* and *have to*). But *may* and *can* (and *must*/*have to*) express the same thing and that thing (whatever it is) is a thing we can sign-up for negating.) This seems dodgy: we're now wheeling in another explaining-away to save our original explaining-away. And the new explaining-away won't help once we embed the (apparent) negated conditional in an even larger environment:

(14) If there is no God, then it's not so that if I pray my prayers will be answered.

So the implicature defense needs work, including saying how the speech act of denial compositionally mixes with the assignment of semantic values for the rest of the language.

4 Variably Strict Conditionals

Assume that indicatives do not have the truth conditions of material conditionals. They say more. What can that more be? Stalnaker (1975) argues that indicatives share a core semantics with counterfactuals. Both kinds of conditionals, he says, are *variably strict* conditionals, but indicatives are governed by additional pragmatic mechanisms that counterfactuals aren't subject to. The variably strict semantics allows indicatives to say more than their horseshoes, and the pragmatic mechanisms explain why certain patterns that *aren't* entailments nevertheless seem so compelling.

Start with orderings over the set of possibilities: for each w, assume there is a (connected, transitive) relation $\leq_w$ recording relative similarity or closeness between worlds. There are two points of interaction between contexts and the ordering: (i) what ordering is relevant is contextually determined and (ii) Stalnaker posits a substantive pragmatic constraint linking contextually relevant possibilities and the ordering. Let's simply model contextually relevant information determined by contexts as functions from worlds to sets of compatible worlds.

Variably Strict Semantics Indicatives are variably strict conditionals:

Assume for any worlds w, v and proposition X, that $\leq_w$ is such that:

i. (Centering) w is minimal in $\leq_w$: if $v \leq_w w$ then $w = v$

ii. (Limit) X has at least one $\leq_w$-minimal world

iii. (Uniqueness) X has no more than one $\leq_w$-minimal world

Then $[\![(\textit{if } P)(Q)]\!]^{c,w} = 1$ iff Q is true at the $\leq_w$-minimal P-world.

This is Stalnaker's (1968) set-up. (Lewis's (1973) version differs by making neither the Limit Assumption nor the Uniqueness Assumption. The pragmatic mechanisms for indicatives that Stalnaker develops are independent of whether the core semantics is Stalnakerian or Lewisian. I'll make the Limit Assumption and be willfully sloppy about whether what's required is the closest antecedent world or the set of closest antecedent worlds.)

This predicts that indicatives say more than their corresponding horseshoes. The paradoxes of material implication won't be reproduced: the inferences in (11) are non-entailments for a variably strict conditional. And while the material conditional validates antecedent strengthening and contraposition, variably strict conditionals don't:

(15) a. If there is sugar in the coffee, it tastes sweet.
$(\mathit{if}\ P)(Q)$

b. #So: If there is sugar and diesel oil in the coffee, it tastes sweet.
$(\mathit{if}\ (P \wedge R))(Q)$

(16) a. If it rains, it won't pour.
$(\mathit{if}\ P)(\neg Q)$

b. #So: If it pours, it won't rain.
$(\mathit{if}\ Q)(\neg P)$

The nearest P-worlds need not include the nearest $(P \wedge R)$-worlds, so (15a) won't entail (15b). Similarly, the nearest rainy worlds can all be drizzly worlds, in which case (16a) is true. Still, the nearest pouring-worlds will be rainy worlds and so (16b) will be false.

Since indicatives are stronger than horseshoes, the direct argument is also classified as a non-entailment:

Counterexample. Suppose w is a $(\neg P \wedge Q)$-world and v is a $(P \wedge \neg Q)$-world and that v is the closest P-world to w. □

Why are instances of it — like (6) — so compelling? Stalnaker's (1975) answer is that it is a *reasonable inference*. Put it this way:

Reasonable Inference Suppose P is successfully asserted (at w in c) and c' is the resulting posterior context. P, *so* : Q is a reasonable inference iff Q is accepted in c'.

This is not a semantic property. It depends on what speakers are up to in asserting various things. In order to be successfully asserted, a premise has to first be felicitously asserted. In order for a conclusion to be accepted in a context, it has to get a clean bill of pragmatic health. Since *or*-to-*if* begins with a disjunctive premise and ends with a conditional conclusion, we need two additional pieces of information. Begin with the disjunctive premise: in order for a disjunction to be felicitously asserted in a context either disjunct might be true without the other. That seems plausible.

The constraint on indicatives is more substantive. In order to use an indicative conditional (owing perhaps to their epistemic connection) there is an additional requirement that the selected antecedent world(s) must be compatible with the context, assuming the antecedent is:

Pragmatic Constraint If P is compatible with the context c, then the $\leq_w$-minimal P-world(s) is compatible with c.

This is motivated this way. First: indicatives are happiest being asserted when their antecedents might, in view of the context, be. If you want to say something conditional on P in a context in which it is settled that $\neg P$, then the counterfactual is what you need to reach for. Second: in asserting indicatives, you are trying to say something about the possibilities compatible with the context. So the worlds relevant to whether an indicative is true had better be compatible with the context. This plus the plausible requirement for disjunctions guarantees that the direct argument is a reasonable inference:

Argument. Suppose an assertion of (6a) is successful (felicitous and accepted) at w in a context c. So $c(w)$ contains some $(P \wedge \neg Q)$-worlds and some $(\neg P \wedge Q)$-worlds. Since the disjunction is then accepted, we eliminate worlds from $c(w)$ where it isn't true. In the posterior $c'(w)$, there are only $(P \vee Q)$-worlds. Some are $(P \wedge \neg Q)$-worlds and some are $(\neg P \wedge Q)$-worlds. Now consider the indicative (6b): $(\mathit{if}\ \neg P)(Q)$. Its antecedent is compatible with c' (there are $\neg P$-worlds in $c'(w)$). Thus the pragmatic constraint ensures that the ($\leq_w$-)closest $\neg P$-world(s) are also in $c'(w)$. But there aren't any $(\neg P \wedge \neg Q)$-worlds in $c'(w)$. So the closest $\neg P$-worlds will be Q-worlds. So the indicative is accepted in c'. □

That's why, according to Stalnaker, *or*-to-*if* instances — though invalid — are so compelling.

A variably strict semantics also fails to classify pairs of conditionals like (7) as mutual entailments: whatever the merits of import/export, given the variably strict semantics, it ain't a validity. That's because the nearest Q-world to the nearest P-world to w need not be the same as the nearest $(P \wedge Q)$-world to w, even when both P and Q are compatible with the context. In the case of the direct argument, there is an independently plausible felicity-requirement on the disjunctive premise — that in the prior context either disjunct may be true without the other — that does real work in making sure the posterior context will be one in which the indicative is accepted. For import/export it's not obvious what such an extra, independently plausible felicity-requirement on the conditional premises (in either direction) would look like. And so it's not obvious how to use the concept of reasonable inference to explain why import/export instances — though invalid — are so compelling.

5 NTV and Conditional Assertion

The boundedness argument can be seen as an argument that there is no good way of assigning truth conditions to indicatives: the only options come up short either by giving up something important or by saddling us with the horseshoe theory. Gibbard (1981) sees it that way and sees it as reason conclude, with Adams (1975), that indicative conditionals don't have truth conditions at all: they do not report conditional information but are a means of expressing conditional belief. Edgington (1995, 2009) and Bennett (2003) (among others) follow them to this "N(o)T(ruth)V(alue)" conclusion. (This view sounds like it denies that indicatives have truth values. In that it does not disappoint. But the spirit is to deny something more. We've assumed that conditionals have the same sort of semantic value that other bits of declarative language have. NTVers deny that indicative conditionals have or traffic in the same kind of semantic unit of exchange that other declarative parts of the language do: it's not truth and it's not some richer-than-than-truth thing either.)

Another argument against truth conditions, this one due to Edgington (2009: §2): truth conditions come in two flavors (truth-functional and non-truth-functional) and neither seems right for indicatives. Truth-functional truth conditions are out because the horseshoe is out. That leaves non-truth-functional ones. Saying that the truth conditions for $(\textit{if } P)(Q)$ are non-truth-functional means that the truth values of P and Q at w don't fix the truth value of $(\textit{if } P)(Q)$ at w. So if P is false at w and Q is true at w then sometimes $(\textit{if } P)(Q)$ is true at w and sometimes it isn't. That's *variability*. But we have the *or*-to-*if* inference: if all the information you have is that $\neg P \vee Q$ then that is *always* sufficient for $(\textit{if } P)(Q)$. This is *uniformity*. Edgington says variability and uniformity are on a collision course and no non-truth-functional theory can be squared with this. If all that is right, we are out of options truth conditions-wise. (I myself think this isn't all right, since for all that's been said it's possible that the variability be variability in truth value at a given world between contexts and the uniformity be uniformity across worlds compatible with a given context (Gillies 2009: §4).)

A third argument is based on hedging. Lewis (1976, 1986) showed that on pain of triviality there are no conditional propositions such that their probability of truth always equals the conditional probabilities of their consequents given their antecedents. (The basic result has been fortified and extended: see, e.g., Hájek (1994). Bennett (2003: §25–31) surveys some of the main territory.) There is a connection that needs explaining:

(17) a. Probably, if my team doesn't sign some big players they won't win.
b. If my team doesn't sign some big players they won't win.

The hedged (17a) seems to say that the probability that my team will continue their losing ways, conditional on them continuing their frugal/mismanaging ways, is high enough. I don't think that they definitely won't win if they don't make some big moves, but I think that's likely (enough). So the degree to which I am signed up for the bare (17b) seems to track what (17a) says. Suppose B is a probability function representing degrees of belief or assertability or whatever. That's some reason to sign-up for this:

The Equation $B((\textit{if } P)(Q)) = B\text{-plus-}P(Q)$

We haven't yet said anything about how we arrive at this posterior probability B-plus-$P(Q)$. Maybe it's through conditionalization, maybe not. The Equation implies:

Linearity If B is a mixture of B_1 and B_2 then B-plus-P is a mixture of B_1-plus-P and B_2-plus-P.

Proof. Assume B is a mixture of B_1 and B_2. Consider any P and Q. So $B((\textit{if } P)(Q))$ is a mixture of $B_1((\textit{if } P)(Q))$ and $B_2((\textit{if } P)(Q))$. But since we're assuming The Equation that means B-plus-$P(Q)$ is a mixture of B_1-plus-$P(Q)$ and B_2-plus-$P(Q)$. □

Gärdenfors (1982) showed that Linearity is at odds with conditionalization. So The Equation implies that the *plus* in B-plus-P can't be conditionalization:

Observation. The Equation is not compatible with conditionalization.

One way of seeing things: Gärdenfors showed that Linearity exactly characterizes changing the B's by *imaging* (imagining and conditionalization can agree only at the boundaries). Another way: conditionalization implies *conservativism*: whenever $B(Q) = 1$ and $B(P) > 0$, then $B_P(Q) = 1$. Linearity rules that out (modulo triviality). Lewis's (1976) main triviality results then follow as corollaries.

(All of this is replicated if we think of the B's as sets of full beliefs. The qualitative counterpart of The Equation:

(RT) $(\mathit{if}\ P)(Q) \in B$ iff $Q \in B * P$

Again, no restrictions on how we arrive at the posterior $B * P$. (RT) implies the qualitative counterpart to Linearity:

(Mon) $B_1 \subseteq B_2$ implies $B_1 * P \subseteq B_2 * P$.

(Suppose $Q \in B_1 * P$. Then $(\mathit{if}\ P)(Q) \in B_1$ and so $(\mathit{if}\ P)(Q) \in B_2$. So $Q \in B_2 * P$.) Just as Linearity is at odds with conservativity, (Mon) is at odds with it's counterpart "preservation" ($\neg P \notin B$ implies $B \subseteq B * P$) (Gärdenfors 1988; Segerberg 1989).)

There are nearby problems, too. I'm dead certain that the Cubs didn't win it all this year, but think it's possible that they won at least 70 games (at a certain point, you have to just quit keeping track).

(18) a. ??Probably, if the Cubs won at least 70 games, then they won it all this year.
b. If the Cubs won at least 70 games, then they won it all this year.

Given what I know, there's no hedge low enough for (18a) to be OK and no degree of belief small enough for me to sign up for the bare (18b) even to that minimal degree. Bradley (2000) argues that treating indicatives as things expressing propositions — subject to the same probability hedges that non-conditional stuff is — means having to say that it is sometimes OK to give positive weight to a conditional and its antecedent even though you give no weight at all to its consequent.

So if indicatives express propositions of the normal sort and scope under probability operators, we have trouble. NTVers say so much the worse for the idea that indicatives express propositions, have truth conditions, and determine truth-values at all. The reason you sign-up for an indicative just to the degree that you conditionally sign-up for its consequent given its antecedent is the straightforward reason that indicatives are vehicles expressing (not representing) such conditional signings-up.

NTVers don't have to worry about triviality under hedges, but this is an open route only if they can explain why inferences involving indicatives seem so compelling — it can't be because they are entailments, the preservation of the main semantic value from premises to conclusion or something like that. Here they have something to say: our intuitive judgments are tracking Adams's (1975) notion of *probabilistic validity* — an argument is probabilistically valid iff the uncertainty of its conclusion is more than the sum of the uncertainties of its premises (for an introduction see Adams (1998)). Modus ponens and modus tollens are probabilistically valid, the paradoxes of material implication (11) aren't. But neither is conditional proof. And without saying something extra and special-purpose about embedded conditionals, neither is import/export.

Embedding is another worry for NTVers. That is because they have to worry about what we're doing when we issue ordinary indicatives. Conditionals aren't, they say, for asserting. They are vehicles of conditional assertion. So in uttering an indicative $(if\ P)(Q)$ (at w, in c), we are conditionally asserting Q-on-P in c at w. That affects $c(w)$ in the same way that flat-out asserting Q affects $c(w)$-plus-P. But for this to fly, Q has to be the sort of thing that can be flat-out asserted in the first place. That means NTVers have revisionary work to do to explain away apparently negated conditionals like (12) and revisionary work to do to explain away the apparently right-nested indicatives featured by import/export. They also face hard choices when it comes to epistemic modals. Consider:

(19) a. If Red isn't in the box, Blue must be.
 b. If Red isn't in the box, Blue might be.

Either we extend the NTV thesis to the modals or not. If so, then embeddings like (19) are prima facie ruled out. If not, we cut ties between the modals and conditionals. Sometimes NTVers say that embedding facts go the other way: since indicatives don't always freely embed (they are tough to negate and it's tough to left-nest them) this suggests that they don't express propositions. This is too fast. Some lexical items have syntactic restrictions on embedding that have nothing to do with their semantics. We've seen this already: *might* and *must* resist embedding under negation (and resist embedding under deontic modal operators) even though *can* and *have to* aren't so fickle.

6 Epistemic Operators

Set aside NTV views. Interaction between indicative conditionals and epistemic operators (hedges and modals alike) is still tricky. Examples:

(20) a. If the gardener isn't guilty, the butler must be.
 b. If he didn't tell Harry, he probably told Tom.
 c. If Carl is here, then presumably Lenny is here.
 d. If the Cubs get good pitching and timely hitting, they might win.

These operators seem to occur embedded, giving us (apparently) instances of this:

(21) $(\textit{if } P)(\textsc{operator}\ Q)$

This takes *if*-of-English to contribute a conditional relation as its meaning, and further says that that conditional relation holds between P-worlds and worlds where OPERATOR Q is true. The trouble is there seems to be no such conditional relation. Grant some minimal assumptions about what conditional relation between (relevant) antecedent-worlds and consequent worlds an $(\textit{if } \cdot)(\cdot)$ picks out (at w in c). (For instance, the relation has to be idempotent and right upward monotonic and it has to care about consequents.) Then:

Observation. The only conditional relation that $(\textit{if } P)(Q)$ can express between the relevant antecedent worlds and consequent worlds is $\subseteq$.

For proofs (of slightly different versions of this), see van Benthem 1986; Veltman 1985; Gillies 2010.

Now grant some minimal assumptions about the *if*-relevant worlds at w: (i) w is always relevant to an indicative $(\textit{if } P)(Q)$ at w; (ii) the *if*-relevant worlds (in c at w) are compatible with c. And grant that modals are quantifiers over worlds compatible with the context: a *might* at w in c is an existential quantifier over $c(w)$ and *must* is the dual universal quantifier.

To highlight the trouble: I have lost my marbles and know that one and only one — either Red or Yellow — is in the box.

(22) a. Red might be in the box and Yellow might be in the box.
$\textit{might}\, P \wedge \textit{might}\, Q$
b. If Yellow isn't in the box, then Red must be.
$(\textit{if } \neg Q)(\textit{must}\, P)$
c. If Red isn't in the box, then Yellow must be.
$(\textit{if } \neg P)(\textit{must}\, Q)$

These are all true. But it seems they can't all be true together if $(\textit{if } \cdot)(\cdot)$ means $\subseteq$. No matter what semantics for indicatives we pick, the sentences in (22) are incompatible. While it's a lamentable fact that my marbles are lost, it's still a fact not an impossibility.

Proof Sketch. Suppose otherwise and that we have just two worlds compatible with the context, w and v. Look at w: it's either a $\neg Q$-world or a

$\neg P$-world. Suppose it's a $\neg Q$-world. Since $(if\ \neg Q)(must\,P)$ is true at w in c all the *if*-relevant $\neg Q$-worlds are worlds where *must* P is true. But w must be one of the relevant *if*-relevant worlds and it's a $\neg Q$-world, too. So it's a world where *must* P is true. So $c(w)$ has only P-worlds compatible with it. But *might* Q is true at w! Mutatis mutandis if w is a $\neg P$-world. (This is equally an apparent argument against modus ponens for indicatives, assuming that entailment is simple preservation of truth(-at-a-point).) □

Ah, the problem is that we got the scope relations wrong! Epistemic operators, you say, scope over the indicatives in environments like (20). Replace (21) with

(23) OPERATOR $(if\ P)(Q)$

This is no better, really. For one thing if OPERATOR is a probability hedge, triviality looms. (There are other reasons, but they involve questions about whether we decide to be *egalitarian* (all worlds compatible with the context are created equal when it comes to saying which are *if*-relevant) or *chauvinistic* (not egalitarian). Those questions are left open given what we've said, but when paired with the widescoping strategy in (23), each leads to different trouble (Gillies 2010: §6).) We appear to be out of choices, scope-wise.

7 Restrictor View

This trouble is a lot like some trouble Lewis (1975) saw for *if*s in quantificational environments liken these:

(24) $\left\{\begin{array}{r}\text{Always}\\ \text{Sometimes}\\ \text{Seldom}\end{array}\right\}$ if a farmer owns a donkey, he beats it.

What single connective could *if* contribute in each of these? Maybe something iffy would work if the adverb is the universal *always*, but conjunction would be better if its an existential like *sometimes* and neither looks good for *seldom*. The *if* has the job of restricting the quantifiers. Lewis's (1975) conclusion: the restricting job isn't one that a conditional operator can do. So sentences like those in (24) are not instances of a conditional operator plus an adverb of quantification. Instead, he said, the *if*s are a *non-connective* whose only job is to mark an argument slot for the adverb of quantification.

Kratzer's idea (developed in, e.g., Kratzer 1981, 1986, 1991, 2010) is that this holds not just for *if*s under adverbs of quantification but for *if*s across the board. The thing all *if*s do is restrict operators. So they aren't iffy. The choice between (21) and (23) is not what we are after. Instead what we have is something like this:

Restrictor Analysis Indicative conditionals are restricted operators with logical forms like this:
Quantifier/Operator + *if-clause* + consequent clause
$\text{OPERATOR}(P)(Q)$

When the operator is a modal like *must* or *probably* or *might*, the job of the *if*-clause is to restrict the domain over which the modal quantifies. If there is no overt operator, then since *if*s restrict operators, there must be a covert (necessity) one.

This picture is general and powerful. (For more on the restrictor view, see von Fintel (2011) and the references therein.) It is, for instance, easy to see why indicatives go in for import/export. It is also easy to see how and why conditionals (which is to say "conditionals") and operators — like the unrestricted *might*s in (22) — interact. This is true for fancier modals and hedges, too:

(25) a. If he's a Quaker, he's presumably a pacifist.
b. If the bet is on odd, it's probably a loser.
c. If the coin is fair, then the probability of heads is $\frac{1}{2}$.

Each of these expresses some restricted hedge: that he's presumably a pacifist (given he's a quaker), that the bet won't be won (given it's on odd), that the probability of heads is $\frac{1}{2}$ (given the coin is fair). The hedges — *presumably*, *probably*, and x-*probably* — are modal operators. Treating *if*s as restrictors means that when we look at environments like these we shift our attention to the operators involved. Getting straight about them is how we get straight about the conditionals in which they occur. Though our current topic is indicatives and not hedges like these, I'll just briefly mention two routes to modeling their contribution that fit hand/glove-wise with the restrictor view of *if*s.

Route one: introduce a qualitative ordering between possibilities and eek truth-conditions for a hedge at w in c out of this. This is the route that Kratzer takes. (There are a lot of choices for the qualitative structure used and choices for how to connect it to the hedges. Yalcin (2010) surveys some of the choices.) Route two: introduce a more fine-grained body of information at hand determined by a context c. In particular: allow contexts to provide probability measures over the possibilities compatible with them. (Here, too, there are lots of choices.) Each route ties hedges to a certain body of information and that body of information can be restricted in pretty straightforward ways: all the choices allow for a well-understood way of "updating" information that bears a systematic and well-understood connection to conditionalization. (For more on the choices for representing the needed uncertainty the hedges trade on, and how those ways of modeling it go in for updating, see Halpern 2003.)

8 Re-Inventing Conditional Connectives

There are still reasons to explore properly iffy operators that carry conditional meanings. One reason: the charge is that the restricting job that *if*s unquestionably do is a job that can't be done by any conditional operator. Does the charge stick? It's worth knowing. (Another reason: the way hedged *if*s (and their partnering iffed hedges) behave in conversation seems to require a conditional connective to figure in the story. (The history of the problem is a little involved, but see von Fintel & Gillies (2011).) That would be surprising not least because Lewis-style triviality results seem to push exactly the other way. But the jury is currently still out on this particular bundle of puzzles.) The re-inventions sketched here also have an answer on offer for what the extra bit is that indicatives say over and above their corresponding horseshoes (naturally enough the bit the different re-inventions offer do differ).

One way that *if* can express a uniform conditional meaning while still doing its restricting job is for the conditional meaning to be a gappy one — gaps for when things aren't as the *if*-clause says — and for our story of embedding operators to be sensitive to those gaps. The idea is Belnap's (1970) and it has been revived and pushed by Huitink (2009). (Lewis knew this was a way out, but dismissed it.)

Here is the bare bones Belnap view: a conditional $(if\ P)(Q)$ (in c at w) says pretty much the same as Q does (in c at w) provided P is true. Otherwise, it says nothing. Two ways to try to make this go: "boring" and "interesting". The boring way says sameness is sameness of *truth-value*, interesting that it is sameness of *semantic value*. Interestingly, it is the boring way that seems more promising. This is what Huitink does.

Belnap Conditional Indicatives express gappy propositions:

$$[\![(if\ P)(Q)]\!]^{c,w} = \begin{cases} [\![Q]\!]^{c,w} & \text{if } [\![P]\!]^{c,w} = 1 \\ \text{undefined} & \text{otherwise} \end{cases}$$

This treats $(if\ P)(Q)$ like the horseshoe when P is true but diverges from it in just the cases that the horseshoe theory goes wild. But it doesn't by itself meet the challenge of getting the restricting behavior right. If the operator that needs restricting is out-scoped by the *if* we will get the wrong and unrestricted truth-conditions and if the operator does the out-scoping we will *still* get the wrong and truth-conditions. The fix is easy. First: widescope the operators. Second: reign in their quantificational domains to survey only worlds/cases/situations/whatever at which the embedded sentence gets a truth-value. That's it.

There is another way to re-invent conditional connectives based on Ramsey's (1929/1990) suggestion. The Ramsey test is a recipe for when to *accept* a conditional: you accept $(if\ P)(Q)$ in a state B iff Q is accepted in the subordinate state got by taking B and adding the information that P to it. But we want a story about what *if*s mean, not so much about when they are/ought to be accepted. Again the fix is easy: $(if\ P)(Q)$ in a context c says that all (relevant) P-possibilities are possibilities at which Q is true. What is the context relevant for checking at those P-possibilities whether Q is true? The subordinate context got by taking c and adding the information that P to it. Making the straightforward choice for how the adding goes (zoom in to the possibilities where P is true):

Shifty Conditional Indicatives express shifty propositions:

$$[\![(if\ P)(Q)]\!]^{c,w} = 1 \text{ iff } c(w) \cap [\![P]\!]^{c} \subseteq [\![Q]\!]^{c+P}$$

$$\text{where } c + P = \lambda v.c(v) \cap [\![P]\!]^{c}$$

If we narrowscope the relevant operators, leaving their plain vanilla semantics intact, this is enough to get the restricting behavior of *if*-clauses to mesh with them expressing a genuine conditional connective.

This re-invention is equivalent to dynamic semantic accounts that types all sentences in the language as programs or instructions for changing the context. (The classic references: Groenendijk & Stokhof (1991); Veltman (1996).) Intuition: a program means what it does, and what it does depends on what things are like — the state you are in — when you execute it. So, a program's content is a relation between prior states and corresponding posterior states.

Dynamic semantics treats all sentences that way: their contents are the characteristic changes that successful assertions of them induce. A simple example:

Dynamic Semantics for Propositional Logic Take a state (context) s to be a set of worlds.

i. $s[P] = \{w \in s\colon\ P \text{ is true in } w\}$ for atomic P's

ii. $s[\neg P] = s \setminus s[P]$

iii. $s[P \wedge Q] = s[P][Q]$

Here the relation is in fact a function $[\cdot]$ from states to states (read it postfix). So (successfully asserting) atomic sentences tell us to throw away worlds where they aren't true, negations tell us to throw out what would survive an update with the things negated, and conjunctions tell us to process things in order. The point of each these instructions is to have a non-null upshot.

Not all instructions have that aim. You say to me: *Check whether the game is on.* You're not instructing me to change anything game-wise or TV-wise but instructing me to see whether the state we are in has a certain property. It's a *test* program. The dynamic idea is that indicatives are Ramsey-inspired test instructions:

Dynamic Conditional Indicatives express test programs:

$$s[(\mathit{if}\ P)(Q)] = \{w \in s\colon\ s[P][Q] = s[P]\}$$

Let's say that a sentence P is true in state s iff the information that P is already present in s. That is: P is true in s iff $s[P] = s$. Then we can put things this way: an indicative $(\mathit{if}\ P)(Q)$ is true in a state s iff Q is true in the subordinate state $s[P]$ got by taking s and adding the information P to it. That is very Ramseylike.

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