On Truth-Conditions for *If* (But Not Quite Only *If*)

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On Truth-Conditions for *If* (But Not Quite Only *If*)

The Folklore
(1) If Carl is away, then if Lenny is away then Sector 7G is empty

(2) If Carl is away and Lenny is away, then Sector 7G is empty.
On Truth-Conditions for *If* (But Not Quite Only *If*)

The Folklore

**How We’d Love to Assign Truth-Conditions to *If***

- Indicative conditionals are bounded from above by strict implication
  - must be true when their antecedents entail their consequents
- Indicative conditionals are bounded from below by material implication
  - must be false when their antecedents are true but consequents are false
- Pairs of indicatives like (1) and (2) are equivalent
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The Folklore

But We Can’t – Because then *If* $= \Box$

$:=(
But We Can’t – Because then $If = \supset$
On Truth-Conditions for *If* (But Not Quite Only *If*)

The Argument

The Argument
Preliminaries

Fix a (propositional) language $L$: atoms, $\neg$, $\land$

$L$ also has binary connective $(if \cdot)(\cdot)$

Let $p, q, r, \ldots$ range over the fragment with no if's

Let $P, Q, R, \ldots$ range over arbitrary sentences of $L$
On Truth-Conditions for *if* (But Not Quite Only *if*)

The Argument

**Folklore (Classic)**

(3) \((if \ p \supset q)((if \ p)(q))\)

- this is everywhere true
- it entails \((p \supset q) \supset (if \ p)(q)\)
- so \(p \supset q\) entails \((if \ p)(q)\)
Folklore (Classic)

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The Argument

**Folklore (Classic)**

\[(3) \quad (if \ p \supset q)((if \ p)(q))\]

- this is everywhere true
- it entails \((p \supset q) \supset (if \ p)(q)\)
- so \(p \supset q\) entails \((if \ p)(q)\)
Folklore (Remix)

Upper  If $p$ entails $q$ then $(if \; p)(q)$ is true at $i$

Lower  If $(p \land \neg q)$ is true at $i$ then $(if \; p)(q)$ is false

Import  $(if \; (p \land q))(r)$ entails $(if \; p)((if \; q)(r))$

Then  $(if \; p)(q)$ is true at $i$ iff either $p$ is false or $q$ is true
Suppose $p$ is false

- $(\neg p \land p)$ entails $q$
- $(if (\neg p \land p))(q)$ is true at $i$
- $(if \neg p)((if p)(q))$ is true at $i$
- $(if p)(q)$ can't be false at $i$

$\therefore (if p)(q)$ is true if $p$ is false
Suppose $q$ is true

- $(q \land p)$ entails $q$
- $(\text{if } (q \land p))(q)$ is true at $i$
- $(\text{if } q)((\text{if } p)(q))$ is true at $i$
- $(\text{if } p)(q)$ can't be false at $i$

$\therefore (\text{if } p)(q)$ is true if $q$ is true
On Truth-Conditions for *If* (But Not Quite Only *If*)

First Version
Schoolyard Version of the Ramsey Test (Belief Version)

\((if \ P)(Q)\) is acceptable in belief state \(B\) iff

- \(B'\) is the result of “adding” the information that \(P\) to \(B\)
- \(Q\) is accepted in \(B'\)
Strict Conditional Story

Turn the Ramsey Test into a way of assigning truth-conditions

- Sentences get truth values at an index (world) wrt a context
- $\llbracket P \rrbracket_{c,i}^c = 1$ iff $P$ is true at $i$ in $c$ iff $i \in \llbracket P \rrbracket^c$
- for non-iffy bits context is idle
Strict Conditional Story

Turn the Ramsey Test into a way of assigning truth-conditions

- Contexts determine sets of relevant possibilities
- *If*s are Ramseyan strict conditionals over such sets
- Meaning of *if* is context-dependent (*≠ unruly*)
Contexts

Q What’s a context?

A I’m not sayin’

Contexts are whatever they need to be to do the job assigned to them by the strict conditional story.
Contexts

Q  What’s a context?

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Contexts – I Meant What I Said

- Contexts are “realistic” – else they couldn’t do the job assigned to them

- They aren’t necessarily “Stalnakerian” either – I’m not saying it’s not our presuppositions that determine them

- Contexts are whatever they need to be to do the job assigned to them by the strict conditional story
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First Version

**Contexts – Well-Behavedness**

- $i \in c(i)$

- if $j \in c(i)$ then $c(i) \subseteq c(j)$

Together they guarantee

- if $j \in c(i)$ then $c(i) = c(j)$
Contexts – Well-Behavedness

- $i \in c(i)$

- if $j \in c(i)$ then $c(i) \subseteq c(j)$

Together they guarantee

- if $j \in c(i)$ then $c(i) = c(j)$
My Version of the Schoolyard Version of the RT

\((\textit{if } P)(Q)\) is \textbf{true} at \(i\) \textit{wrt} \(c\) \textit{iff} \ldots

\ldots All \(P\)-possibilities in \(c(i)\) are possibilities at which \(Q\) is true
My Version of the Schoolyard Version of the RT

But truth depends on index and context

Q What context is relevant for seeing if $Q$ is true at $P$-possibilities in $c(i)$?

A Ramseyan context $c$-plus-information-that-$P$, $c + P$
My Version of the Schoolyard Version of the RT

But truth depends on index and context

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**Q** What context is relevant for seeing if $Q$ is true at $P$-possibilities in $c(i)$?

**A** Ramseyan context $c$-plus-information-that-$P$, $c + P$
Two Jobs for *if*-clause

- **Restricts** set of indices throughout which we check for consequent’s truth

- **Contributes** to relevant context for figuring out – at such an index – whether consequent is true
Strict Conditional, v.1

- \( [(if \ P)(Q)]^{c,i} = 1 \) iff \( c(i) \cap [P]^c \subseteq [Q]^{c+P} \)

- \( c + P = \lambda i. c(i) \cap [P]^c \)
On Truth-Conditions for *If* (But Not Quite Only *If*)

First Version

**Propositions (Northern Semantics)**

- $[(if \ P)(Q)]^{c,i} = 1 \iff c(i) \cap [P]^c \subseteq [Q]^{c+P}$

- $c + P = \lambda i. c(i) \cap [P]^c$
Upper  Universal quantifiers are $\downarrow$monotone

Lower  Since $i$ is always relevant to a conditional at $i$, if it is a $(p \land \neg q)$-world then not all $p$-worlds in $c(i)$ are $q$-worlds
On Truth-Conditions for *If* (But Not Quite Only *If*)

First Version

Import

Suppose $\llbracket (if \ (p \land q)) (r) \rrbracket^{c,i} = 1$

**Show** All $p$-worlds in $c(i)$ are worlds at which $(if \ q)(r)$ is true wrt $(c + p)$

Take any $p$-world $j$ in $c(i)$

$\llbracket (if \ q) (r) \rrbracket^{c+p,j} = 1$ iff $(c + p)(j) \cap [q] \subseteq [r]$

iff $(c(j) \cap [p]) \cap [q] \subseteq [r]$

iff $(c(i) \cap [p]) \cap [q] \subseteq [r]$

iff $\llbracket (if \ p \land q) (r) \rrbracket^{c,i} = 1$
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Suppose $\mathbb{[(if \ (p \land q))\ (r)]}^{c,i} = 1$

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$\mathbb{[(if \ q)(r)]}^{c+p,j} = 1$ iff $(c + p)(j) \cap [q] \subseteq [r]$

iff $(c(j) \cap [p]) \cap [q] \subseteq [r]$

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iff $\mathbb{[(if \ p \land q)(r)]}^{c,i} = 1$
On Truth-Conditions for If (But Not Quite Only If)

First Version

Import

Suppose \( [(if (p \land q))(r)]^{c,i} = 1 \)

Show All \( p \)-worlds in \( c(i) \) are worlds at which \( (if q)(r) \) is true wrt \( (c + p) \)

Take any \( p \)-world \( j \) in \( c(i) \)

\( [(if q)(r)]^{c+p,j} = 1 \iff (c + p)(j) \cap [q] \subseteq [r] \)

\( \iff (c(j) \cap [p]) \cap [q] \subseteq [r] \)

\( \iff (c(i) \cap [p]) \cap [q] \subseteq [r] \)

\( \iff [(if p \land q)(r)]^{c,i} = 1 \)
On Truth-Conditions for *If* (But Not Quite Only *If*)

First Version

**Import**

Suppose $\llbracket (\text{if } (p \land q))(r)\rrbracket^{c,i} = 1$

Show All $p$-worlds in $c(i)$ are worlds at which $(\text{if } q)(r)$ is true wrt $(c + p)$

Take any $p$-world $j$ in $c(i)$

$\llbracket (\text{if } q)(r)\rrbracket^{c+p,j} = 1$ iff $(c + p)(j) \cap [q] \subseteq [r]$  

iff $(c(j) \cap [p]) \cap [q] \subseteq [r]$  

iff $(c(i) \cap [p]) \cap [q] \subseteq [r]$  

iff $\llbracket (\text{if } p \land q)(r)\rrbracket^{c,i} = 1$
On Truth-Conditions for \( \textit{If} \) (But Not Quite Only \textit{If})

--- First Version

**Import**

Suppose \( \lbrack (if (p \land q))(r) \rbrack^{c,i} = 1 \)

**Show** All \( p \)-worlds in \( c(i) \) are worlds at which \((if q)(r)\) is true wrt \((c + p)\)

Take any \( p \)-world \( j \) in \( c(i) \)

\[
\lbrack (if q)(r) \rbrack^{c+p,j} = 1 \iff (c + p)(j) \cap \lbrack q \rbrack \subseteq \lbrack r \rbrack
\]

\[
\iff (c(j) \cap \lbrack p \rbrack) \cap \lbrack q \rbrack \subseteq \lbrack r \rbrack
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\iff (c(i) \cap \lbrack p \rbrack) \cap \lbrack q \rbrack \subseteq \lbrack r \rbrack
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\iff \lbrack (if p \land q)(r) \rbrack^{c,i} = 1
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On Truth-Conditions for \textit{If} (But Not Quite Only \textit{If})

First Version

Import

Suppose \( [ (\text{if} \ (p \land q))(r) ]^{c,i} = 1 \)

Show All \( p \)-worlds in \( c(i) \) are worlds at which \( (\text{if} \ q)(r) \) is true wrt \( (c + p) \)

Take any \( p \)-world \( j \) in \( c(i) \)

\[
[ (\text{if} \ q)(r) ]^{c+p,j} = 1 \iff (c + p)(j) \cap [q] \subseteq [r] \\
\iff (c(j) \cap \lfloor p \rfloor ) \cap [q] \subseteq [r] \\
\iff (c(i) \cap \lfloor p \rfloor ) \cap [q] \subseteq [r] \\
\iff [ (\text{if} \ p \land q)(r) ]^{c,i} = 1
\]
On Truth-Conditions for *If* (But Not Quite Only *If*)
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First Version

if $\neq \supset$

Suppose $c(i)$ has just two worlds, $i$ and $j$

- $i$ is a $(p \land q)$-world
- $j$ is a $(p \land \neg q)$-world
  - $p \supset q$ is true at $i$ in $c$
  - but $(if \ p)(q)$ isn’t

So $(if \ p)(q)$ says more than $(p \supset q)$ does
if $\neq \supset$  

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Intermezzo
Context Dependence ×2

Local  Truth value of some sentences at an index may vary across contexts

Global  Truth values of some sentences may also co-vary within contexts
Context Dependence $\times 2$

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Context Dependence ×2

Local  Truth value of some sentences at an index may vary across contexts

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Edgington Against Non-Truth-Functional Truth-Conditions

- If $p$ is false at $i$ and $q$ true at $i$ then $(\text{if } p)(q)$ can be either
  - non-truth-functionality requires \textit{variability}

- Barest information characterizing $\neg p \lor q$ always supports $(\text{if } p)(q)$
  - \textit{Or-to-If} requires \textit{uniformity}

- So what’s required by \textit{variability} is what’s ruled out by \textit{uniformity}
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Edgington Against Non-Truth-Functional Truth-Conditions

- If \( p \) is false at \( i \) and \( q \) true at \( i \) then \((\text{if } p)(q)\) can be either
  - non-truth-functionality requires **variability**

- Barest information characterizing \( \neg p \lor q \) always supports \((\text{if } p)(q)\)
  - **Or-to-If** requires **uniformity**

- So what’s required by **variability** is what’s ruled out by **uniformity**
Not Quite!

**True** Variability in \((\text{if } p)(q)\) when \(p\) is false and \(q\) true

**Local** But variability can be between contexts, not within any one context

**True** \(\neg p \lor q\) is enough for \((\text{if } p)(q)\) in every context

**Global** But uniformity can live happily with context dependence
On Truth-Conditions for *If* (But Not Quite Only *If*)

Intermezzo

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**True**  Variability in \((if \, p)(q)\) when \(p\) is false and \(q\) true

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**Global**  But uniformity can live happily with context dependence
Variability and Uniformity

Strict conditional story gives uniform but variable truth-values

- $(if\ p)(q)$ false at $i$ in $c$ iff $c(i)$ has any $(p \land \neg q)$-worlds

- So if $c(i)$ characterizes $\neg p \lor q$, there aren’t any such worlds

- And so $(if\ p)(q)$ is true at $i$ in $c$

- Well-behavedness guarantees that holds good for every $j \in c(i)$
Variability and Uniformity

Strict conditional story gives uniform but variable truth-values

- *(if p)(q)* false at *i* in *c* iff *c(i)* has any *(p ∧ ¬q)*-worlds

- So if *c(i)* characterizes ¬p ∨ q, there aren’t any such worlds

- And so *(if p)(q)* is true at *i* in *c*

- Well-behavedness guarantees that holds good for every *j ∈ c(i)*
Variability and Uniformity

Strict conditional story gives **uniform** but **variable** truth-values

- \((if \; p)(q)\) false at \(i\) in \(c\) iff \(c(i)\) has any \((p \land \neg q)\)-worlds

- So if \(c(i)\) characterizes \(\neg p \lor q\), there aren’t any such worlds

- And so \((if \; p)(q)\) is **true** at \(i\) in \(c\)

- Well-behavedness guarantees that holds good for every \(j \in c(i)\)
Variability and Uniformity

Strict conditional story gives \textbf{uniform} but \textbf{variable} truth-values

- \((\text{if } p)(q)\) \underline{false} at \(i\) in \(c\) iff \(c(i)\) has any \((p \land \neg q)\)-worlds

- So if \(c(i)\) characterizes \(\neg p \lor q\), there aren't any such worlds

- And so \((\text{if } p)(q)\) is \underline{true} at \(i\) in \(c\)

- Well-behavedness guarantees that holds good for every \(j \in c(i)\)
On Truth-Conditions for *If* (But Not Quite Only *If*)

Intermezzo

**Shiftiness \times 2**

**Index**  Truth value of some sentences at an index in a context depends on truth value of some constituents at other indices

**Context** Truth value of some sentences at an index in a context depends on truth value of some constituents at other contexts
Shiftiness $\times 2$

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Shiftiness $\times 2, \times 2$

(4) If Carl is at the party, then Lenny is at the party

Index True at $i$ in $c$ iff Lenny is at the party is true at every antecedent-world in $c(i)$

Context Truth of Lenny is at the party is figured wrt to c-plus-info-that-Carl-is-at-the-party
On Truth-Conditions for *If* (But Not Quite Only *If*)

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**Shiftiness** $\times 2, \times 2$

\[(4)\] If Carl is at the party, then Lenny is at the party

**Index** True at $i$ in $c$ iff *Lenny is at the party* is true at every antecedent-world in $c(i)$

**Context** Truth of *Lenny is at the party* is figured wrt to c-plus-info-that-Carl-is-at-the-party
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(4) If Carl is at the party, then Lenny is at the party

**Index**  True at $i$ in $c$ iff *Lenny is at the party* is true at every antecedent-world in $c(i)$

**Context**  Truth of *Lenny is at the party* is figured wrt to $c$-plus-info-that-Carl-is-at-the-party
But Only If

Couldn’t meet Import this way

\[(5) \quad [(if \ P)(Q)]^c, i = 1 \text{ iff } c(i) \cap [P]^c \subseteq [Q]^c\]

Q  Why not?

A  Homework

Hint  Suppose in c either p, q, r are all true or just one is and consider \((if \ p \land q)(r)\) and \((if \ p)((if \ q)(r))\)
On Truth-Conditions for If (But Not Quite Only If)

Intermezzo

But Only If

Couldn’t meet Import this way

$[(\text{if } P)(Q)]^c, i = 1 \text{ iff } c(i) \cap [P]^c \subseteq [Q]^c$

Q Why not?

A Homework

Hint Suppose in $c$ either $p, q, r$ are all true or just one is and consider $(\text{if } p \land q)(r)$ and $(\text{if } p)((\text{if } q)(r))$
But Only If

Couldn’t meet Import this way

(5) \[ [(\text{if } P)(Q)]^{c,i} = 1 \text{ iff } c(i) \cap [P]^c \subseteq [Q]^c \]

Q  Why not?

A  Homework

Hint  Suppose in c either p, q, r are all true or just one is
and consider (if p \land q)(r) and (if p)((if q)(r))
But Only If

Couldn’t meet Import this way

(5) \[ ((\text{if } P)(Q))^{c,i} = 1 \text{ iff } c(i) \cap [P]^c \subseteq [Q]^c \]

Q Why not?

A Homework

Hint Suppose in c either p, q, r are all true or just one is and consider (if \( p \land q \))(r) and (if \( p \))(if \( q \))(r))
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Intermezzo

**But Only If**

Couldn’t meet Import this way

\[(5) \quad [(if \ P)(Q)]^{c,i} = 1 \text{ iff } c(i) \cap [P]^c \subseteq [Q]^c\]

Q  Why not?

A  Homework

**Hint** Suppose in \(c\) either \(p, q, r\) are all true or just one is and consider \((if \ p \land q)(r)\) and \((if \ p)((if \ q)(r))\)
On Truth-Conditions for *If* (But Not Quite Only *If*)

Second Version
Sentences get truth values in contexts, full-stop – take a context $s$ to be a set of worlds now

A sentence is true in a context iff adding that sentence’s information to that context wouldn’t change it

Semantic values are context change potentials – meaning of declarative sentences is like the meaning of programs and instructions
On Truth-Conditions for *If* (But Not Quite Only *If*)

Second Version

Programs $\times 2$

(6) set the value of $x$ to 1

(7) check whether the value of $x$ is 1
On Truth-Conditions for *If* (But Not Quite Only *If*)

 Programs $\times 2$

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(6) set the value of \( x \) to 1

(7) check whether the value of \( x \) is 1
Constraint – CCPs

- \( s[p] = \{ i \in s : i(p) = 1 \} \)
- \( s[\neg P] = s \setminus s[P] \)
- \( s[P \land Q] = s[P][Q] \)
- \( P \) is true in \( s \) iff \( s[P] = s \)
Strict Conditional, v.2

\[ s[(if \ P)(Q)] = \{ i \in s : Q \text{ is true in } s[P] \} \]

Implies

\[ (if \ P)(Q) \text{ is true in } s \iff Q \text{ is true in } s[P] \]
Strict Conditional, v.2

- $s[(if \ P)(\ Q)] = \{i \in s : \ Q \text{ is true in } s[\ P]\}$

Implies

- $(if \ P)(\ Q)$ is true in $s$ iff $Q$ is true in $s[\ P]$
Disrupting Folklore, v.2

**Upper** Same-ish reason

**Lower** Same-ish reason

**Import** If $(\text{if } (p \land q))(r)$ is true in $s$ then

- $r$ is true in $s[p \land q]$ – and so in $s[p][q]$
- $(\text{if } q)(r)$ is true in $s[p]$
- $(\text{if } p)((\text{if } q)(r))$
Disrupting Folklore, v.2

Upper  Same-ish reason

Lower  Same-ish reason

Import If $(if (p \land q))(r)$ is true in $s$ then

- $r$ is true in $s[p \land q]$ – and so in $s[p][q]
- (if q)(r)$ is true in $s[p]
- (if p)((if q)(r))$
Disrupting Folklore, v.2

Upper  Same-ish reason

Lower  Same-ish reason

Import  If \((if (p \land q))(r)\) is true in \(s\) then
- \(r\) is true in \(s[p \land q]\) – and so in \(s[p][q]\)
- \((if q)(r)\) is true in \(s[p]\)
- \((if p)((if q)(r))\)
On Truth-Conditions for *If* (But Not Quite Only *If*)

Second Version

Disrupting Folklore, v.2

**Upper** Same-ish reason

**Lower** Same-ish reason

**Import** If \(( \text{if } (p \land q))(r)\) is true in \(s\) then

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On Truth-Conditions for *If* (But Not Quite Only *If*)

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Second Version

\[ \text{if} \neq \supset \]

**Fact** \( \neg (p \supset q) \) is true in \( s \) iff it is true at every \( i \) in \( s \)

Not so for \( \neg (if\ p)(q) \)

Let \( s \) have just two worlds, \( i \) and \( j \), the first a \( (p \land q) \)-world the second a \( (p \land \neg q) \)-world

**Fact** \( \neg (if\ p)(q) \) is true in \( s \) but \( \neg (p \supset q) \) isn’t

So \( (if\ p)(q) \) says more than \( (p \supset q) \) does
On Truth-Conditions for *If* (But Not Quite Only *If*)

Second Version

\[ \text{if } \neq \supset \]

**Fact** \( \neg(p \supset q) \) is true in \( s \) iff it is true at every \( i \) in \( s \)

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On Truth-Conditions for *If* (But Not Quite Only *If*)

Second Version

\[ \textit{if} \neq \supset \]

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Not so for \( \neg (\textit{if } p)(q) \)

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\textbf{Fact} \( \neg (\textit{if } p)(q) \) is true in \( s \) but \( \neg (p \supset q) \) isn’t

So \( (\textit{if } p)(q) \) \underline{says more} than \( (p \supset q) \) does
Variability and Uniformity, Again

- If $p$ is false at $i$ and $q$ true there are contexts containing $i$ at which $(if \ p)(q)$ are true and some at which it’s false

- Any context characterizing $\neg p \lor q$ is a context at which $(if \ p)(q)$ is true
Variability and Uniformity, Again

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On Truth-Conditions for *If* (But Not Quite Only *If*)

— BOGOF
On Truth-Conditions for *If* (But Not Quite Only *If*)

\[ \text{BOGOF} \]

\((if \ P)(Q)\)

\[ \text{v.1} \quad \text{true at } i \text{ in } c \text{ iff at every } j \in c(i): \left[ Q \right]^{c+P,j} = 1 \]

\[ \text{v.2} \quad \text{true in } s \text{ iff } Q \text{ is true in } s[P] \]

- agree on what indicatives are true
- agree on how they change contexts
- agree on entailments involving them
On Truth-Conditions for *If* (But Not Quite Only *If*)

\[ (if \ P)(Q) \]

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- agree on what indicatives are true
- agree on how they change contexts
- agree on entailments involving them
Agree on Truth

Fact If $s = c(i)$ and $j \in s$, then $(c + P)(j) = s[P]$.

Fact true according to v.1 iff true according to v.2.
Agree on Truth

Fact  If $s = c(i)$ and $j \in s$, then $(c + P)(j) = s[P]$

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Agree on Context Change

Stalnaker Context Change

change brought by \( P \) issued at \( i \) in \( c \) is \( c(i) \cap [P]^c \)

\[
[(if\ P)(Q)]^c = \begin{cases} 
  c(i) & \text{if } c(i) \cap [P]^c \subseteq [Q]^{c+P} \\ 
  \emptyset & \text{otherwise}
\end{cases}
\]
On Truth-Conditions for *If* (But Not Quite Only *If*)

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Stalnaker Context Change

change brought by $P$ issued at $i$ in $c$ is $c(i) \cap [P]^c$

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But Wait

Q What if – as folklore argument requires – the *if* has embedded *if*s? Does \((P \land \neg Q)\) rule out \((if \; P)(Q)\)?

A Not according to Strict Conditional v.1
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Counterexample #1

Suppose

- $p$ is false at $i$ in $c$
- there are $(p \land \neg q)$-worlds compatible with $c$

Then

- $(\neg p)((if\ p)(q))$ is true at $i$ in $c$
- $\neg p$ is true at $i$ in $c$
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Counterexample #1

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Counterexample #2

- Take any $p, q, r$
- suppose it is settled in $c$ that either all of them are true or just one is
- suppose $p$ is true at $i$

Then
- $(if\ p)(((if\ q)(r)))$ is true at $i$ in $c$
- But $(if\ q)(r)$ isn’t
Counterexample #2

- Take any $p, q, r$
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Modus Ponens

An if plus its antecedent brings the truth of its consequent in their wake

Fact

If \((if \ P)(Q)\) and \(P\) are true at \(i\) in \(c\) . . .

. . . then \(Q\) is true at \(i\) in \((c + (if \ P)(Q)) + P\)

. . . and so \(Q\) is true in \((c + P)\)
On Truth-Conditions for *If* (But Not Quite Only *If*)

**Modus Ponens**

An *if plus its antecedent brings the truth of its consequent in their wake*

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Entailment, v.1.0

$P_1, \ldots, P_n$ entails $Q$ iff, for any $i$ and $c$

- if $\llbracket P_1 \rrbracket_{c,i} = 1$ and $\ldots$ and $\llbracket P_n \rrbracket ((c+\ldots+P_{n-2})+P_{n-1}), i = 1$ then $\llbracket Q \rrbracket ((c+\ldots+P_{n-1})+P_n), i = 1$

Fact

$(if \ P)(Q), P \text{ entail-v.1.0 } Q$
$P_1, \ldots, P_n$ entails $Q$ iff, for any $i$ and $c$

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Fact

$(if~P)(Q), ~P \text{ entail-v.1.0 } Q$
Entailment, v.2.0

\( P_1, \ldots, P_n \) entails \( Q \) iff, for any context \( s \)

- if \( P_1, \ldots, P_n \) are all true in \( s \) then so is \( Q \)

Fact

\((if \; P)(Q), \; P \entail-v.2.0 \; Q\)
Entailment, v.2.0

$P_1, \ldots, P_n$ entails $Q$ iff, for any context $s$

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Fact

\((i f \ P)(Q), \ P \text{ entail-v.2.0 } Q\)
Entailment, v.2.1

$P_1, \ldots, P_n$ entails $Q$ iff, for any context $s$

- if $s[P_1] \ldots [P_n] = s'$ then $Q$ is true in $s'$

Fact

$(if \ P)(Q), \ P \ entail-v.2.1 \ Q$
Entailment, v.2.1

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Entailment, v.2.1

\[ P_1, \ldots, P_n \text{ entails } Q \text{ iff, for any context } s \]

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Fact

\( (if \ P)(Q), \ P \text{ entail-v.2.1 } Q \)
Agrree on Entailments

Suppose $s$ and $c$ characterize the same context, $i$ is compatible with $c$, and $c(i) = s$. Consider whether $P_1, \ldots, P_n$ entails $Q$

**Fact** Strict Conditional v.1 + Entailment v.1 =
Strict Conditional v.2 + Entailment v.2.1
On Truth-Conditions for *If* (But Not Quite Only *If*)

Dolci
Embarrassment of Riches?

Doubly shifty strict conditionals are strict conditionals – the former inherit characteristic properties of the latter

- antecedent strengthening
- contraposition
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Antecedent Strengthening

(8) If there is sugar in the coffee, it tastes sweet
??So: if there is sugar and diesel oil in the coffee, it tastes sweet

- suppose all $p$-worlds throughout some set $X$ are $q$-worlds
- then every subset of the $p$-worlds in $X$ are also $q$-worlds
∴ pretty bad news for any strict conditional story
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Strategies

- accept entailment, explain it away on pragmatic grounds
- block entailment outright

Share a common core about felicity of indicatives – they’re happiest when their antecedents are open or live possibilities
On Truth-Conditions for *If* (But Not Quite Only *If*)

--- Dolci

**Pragmatic Route (Ingredients)**

Assume

- *if*s implicate that their antecedents are possible
- entailments – to be happy – must remain entailments when we add the implicatures of their conclusions as explicit premises
  - adding the implicatures shouldn’t make us rethink truth of the premises
Pragmatic Route (Ingredients)

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Pragmatic Route (Antecedent Strengthening)

if there is sugar and diesel oil in the coffee . . . implicates that there might be

(9) There might be sugar and diesel oil in the coffee
??If there is sugar in the coffee, it tastes sweet
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(9) There might be sugar and diesel oil in the coffee
??If there is sugar in the coffee, it tastes sweet
So: if there is sugar and diesel oil in the coffee, it tastes sweet
Pragmatic Route (Contraposition)

(10) If it rains, it won’t pour
    So, if it pours, it won’t rain

(11) It might pour
    If it rains, it won’t pour
    So, if it pours, it won’t rain
On Truth-Conditions for *If* (But Not Quite Only *If*)

Dolci

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Semantic Route (Ingredients)

Assume

- *ifs* presuppose that their antecedents are possible

- a sentence can be true in a context only if its presuppositions are met
On Truth-Conditions for *If* (But Not Quite Only *If*)

— Dolci

**Semantic Route (Amendments)**

**Strict Conditional v.1.1**

- \( [(if \ P)(Q)]^{c,i} \) is defined only if \( c(i) \cap [P]^c \neq \emptyset \)

**Strict Conditional v.2.1**

- \( s[(if \ P)(Q)] \) is defined only if \( s[P] = s' \neq \emptyset \)
Semantic Route (Amendments)

**Strict Conditional v.1.1**

- $[(\text{if } P)(Q)]^{c,i}$ is defined only if $c(i) \cap [P]^c \neq \emptyset$

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- $s[(\text{if } P)(Q)]$ is defined only if $s[P] = s' \neq \emptyset$
Semantic Route (Antecedent Strengthening)

(8) If there is sugar in the coffee, it tastes sweet

So: if there is sugar and diesel oil in the coffee, it tastes sweet

- any context in which premise is true has no sugar-and-diesel worlds

- but that’s what is presupposed by conclusion – and so required if it is to be true
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- any context in which premise is true is one where it might rain and all rainy worlds are not-pouring worlds
- contrapositive presupposes it might pour
- so it can’t be defined – whence can’t be true – in any such context
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On Truth-Conditions for *If* (But Not Quite Only *If*)

Fin