Metal Water: A Metamaterial for Acoustic Cloaking

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Abstract: A generic metamaterial is described that is suitable for making acoustic cloaking devices. The fundamental property is that it mimics the acoustic properties of water, yet can be modified to display anisotropic elastic properties suitable for cloaking. It has the important property that the amount of void space is conserved: a “conservation of cloaked space”.

Acoustic cloaking can, in theory, be achieved using widely different material properties. The reason is that for a given mapping function the transformed version of the original scalar wave equation can be interpreted in terms of different material constitutive theories. In this sense acoustic cloaking is distinct from its counterpart for electromagnetic waves, for which the material corresponding to the transformed Maxwell equations is uniquely defined by the transformation function. The first mechanism proposed for acoustic cloaking was based on the concept of anisotropic density, and a single bulk modulus. Compressible fluids with anisotropic density are physically permissible, by layering homogeneous fluids, for instance. The range of inertial properties required for achieving cloaking is great, however, and could be very difficult to achieve by layering. Significant cloaking of a cylinder was shown to be possible using hundreds of homogeneous fluids to reproduce the smoothly varying properties of the transformed fluids. The number of independent fluids can be reduced to three but only if the three fluids have vastly different properties, e.g. one fluid must have extremely large density, another must have very large compressibility, etc.

An alternative route to acoustic cloaking is possible using pentamode materials (PM). These can be considered as generalizations of compressible fluids that have anisotropic compressibility but isotropic density. As such, they are limiting cases of anisotropic elastic materials in which five of the six Kelvin moduli vanish. The Kelvin moduli are the eigenvalues of the 6x6 matrix of elastic moduli in Voigt notation (suitably represented in tensor form). Anisotropic inertia and pentamode materials are in fact limiting cases of a spectrum of material properties that yield the acoustic equation in the original untransformed coordinates. This material non-uniqueness may be ascribed to a “gauge” freedom in how the transformed particle displacement vector is defined; the full range of acoustically transformed material is described in. The transformed elasticity equations display a similar nonuniqueness which can also be associated with the choice of the displacement gauge relation. Pentamode materials with anisotropic elastic behavior, like fluids with anisotropic density, are not found in nature. Zero elastic moduli implies structural instability, exemplified by the ability of water to flow. But in the case of water, the PM flows from one isotropic state to an identical one. PMs for cloaking cannot flow without change of state, and must have non-zero but small shear rigidity for stability.

Metal Water: Pentamode Material with Small Shear Rigidity

Water is the quintessential PM, however it is isotropic and therefore of no use in cloaking. Yet, any structural material for an acoustic cloak should be able to replicate homogeneous water in an acoustic sense. It must have the same effective compressibility and density as water, and display very small shear rigidity. We describe here a class of metamaterial with these properties, and more importantly, which can be modified to reproduce the PM properties of a cloak. The effective properties hold for wavelengths longer than the microstructure, so that the model is broadband. A metal microstructure in the form of a regular foam is chosen because it has relatively small shear rigidity, while exhibiting the bulk modulus and density of water simultaneously, see Figure 1. The large relative density of metal means that the structure is mostly void space.

An important property of PM cloaking is that the total mass of the cloak must equal the mass of the water in the space occupied by the cloak and the cloaked region. Hence the amount of void space present in MW is preserved under the transformation from homogeneous “water” to the inhomogeneous PM. This “Archimedes principle” amounts to a Principle of Conservation of Cloaking Space, as
shown in Figure 1c. One can think of the original void space in the isotropic MW as micro-cloaked regions. The coordinate transformation has the effect of blowing up one of these while simultaneously shrinking the others.

![Image](a) A schematic of a unit cell. (b) Static deformation under a load used to calculate the effective elastic moduli of the periodic system. (c) The isotropic MW is transformed via the coordinate mapping into a functionally graded cylindrically anisotropic MW structure. The total amount of void in both pictures is preserved: Conservation of Cloaking Space.

The MW design begins with a hexagonal unit cell, depicted in Figure 1a. For the 2D structures considered here, the unit cell comprises thin load bearing struts, with islands of mass at the vertices, Figure 1b. The periodic structure is shown in Figure 1c, both in the original isotropic state on the left, and after transformation to the locally orthotropic structure on the right. The effective elasticity of the macroscopic foam can be calculated asymptotically to leading order in the thickness/length small parameter \( \varepsilon \) as

\[
C = \begin{pmatrix}
c_{11} & c_{12} & c_{16} \\
c_{12} & c_{22} & c_{26} \\
c_{16} & c_{26} & c_{66}
\end{pmatrix} = C_0 \begin{pmatrix}
\alpha & 1 & 0 \\
1 & \alpha & 0 \\
0 & 0 & O(\varepsilon)
\end{pmatrix}, \quad \alpha = \frac{l \cos^2 \theta}{(h + l \sin \theta) \sin \theta},
\]

(1)

By way of comparison, water (or any compressible liquid) is described by stiffness matrix with \( \alpha = 1, c_{66} = 0 \). The matrix of numbers in equation (1) shows the FEM derived effective moduli (in GPa) for the isotropic MW. Note the small value of \( c_{66} \). Numeric results above were obtained using the ANSYS FEM package with MATLAB for data manipulation, utilizing the homogenization theory outlined in\(^\text{12}\). Numerical experiments show that this value of shear modulus produces very small mode coupling and hence scattering of acoustic waves. The parameter \( \alpha \) introduces the necessary anisotropy, which is determined by the unit cell parameters \( \theta \) and \( l/h \) via equation (1). The connection with the cloaking transformation is that it defines \( C_0 \) and \( \alpha \) (and the density) as functions of \( r \). The microstructure is thus directly related to the coordinate mapping.

The presentation will develop these ideas through specific examples of cloak designs. Simulations of acoustic wave scattering will be shown that demonstrate the effectiveness of MW as a candidate metamaterial for not only 2D but 3 dimensional cloaking structures. Results from experiments that are anticipated will be reported if available.

References