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STBC CDMA System Simulation in MIMO Channels with Correlative Model

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Abstract— **In this contribution, Space-Time Block Coding (STBC) Code Division Multiple Access (CDMA) system is simulated to investigate the effect of spatial correlation of Multiple-input multiple-output (MIMO) wireless channels. In practical situations the MIMO antennas are spatially correlated, due to the lack of spacing between them or weak scattering. The correlative model is considered for MIMO channel and simulation results declare that spatial correlation degrades the performance of STBC CDMA system. However, having some information about the channel of each active user at the transmitter, a linear precoder can be exploited, in conjunction with the coding block, to mitigate the effect of correlation in multiuser system which has not been addressed before. In order to design this precoder, the equations of STBC CDMA system, are reconfigured in this paper. Simulation results illustrate that despite the number of interfering users, the anti-correlation precoder could mitigate the performance degradation produced in correlative model.**

Keywords- STBC; CDMA; Correlative model; Precoding; multiuser system

I. Introduction

Recent researches in wireless communications show that there is an increasing demand for more effective and reliable wireless systems. MIMO structure is an effective approach in this area and space-time coding technique can be used in such systems to achieve higher performance, throughput and coding gain. STBC was first introduced by Alamouti [1] and Tarokh [2],[3] in which the code design criteria is calculated under the condition that the transmit and receive antennas are uncorrelated. This may not be accurate in practical situations that MIMO antennas are spatially correlated, due to the lack of spacing between them. The effect of spatial correlation on the MIMO channel capacity has been addressed in [4]. Even though the receive correlation is inconsiderable when there are remarkable scatters around it, the transmit correlation is still an important challenge especially in downlink transmission procedure.

In conventional designs [2],[3] it is assumed that there is no knowledge about the channel (CSI) in the transmitter. However by exploiting the CSI combined with STBC in the transmitter, a precoder can be designed to reduce the problem of spatial correlation [5],[6]. Moreover, as stated in [7], if an STBC is capacity lossless for an i.i.d. MIMO channel, then combining this STBC with a linear precoder is capacity optimal for the channel with CSIT [5].

Several performance measurements such as channel capacity [8] and pairwise error probability (PEP) [7] -which is regarded in this paper- can be exploited for designing the precoder.

On the other hand, the channel correlation effect is addressed in the literature for single user scenarios [10-15]. Working with the 3G systems and 4G candidates, the multiple access techniques should be considered.

Motivated by this fact, in this paper we investigate a multiuser STBC CDMA (code division multiple access) system in correlated MIMO channel. The main contribution of this paper is using an anti-correlation precoder for every separate user to improve the bit error rate (BER) performance of the system in correlated channel model. In order to design this precoder, the equations of STBC CDMA system, are reconfigured in this paper. Despite the number of interfering users, this type of precoder could mitigate the performance degradation caused by correlation. Through various system simulations the effect of correlation and the role of precoding are investigated.

The rest of the paper is organized as follows. Section II describes the correlation model and anti-correlation precoder for the single user case. In section III, STBC CDMA system in spatially correlated channel is considered. The results and comprehensive discussions are presented in section IV. Finally Section V concludes the paper.

II. Correlation Model and Anti-Correlation Precoder for STBC Single User

In general form, we consider a system in which signals are transmitted via N_T transmit antennas and are received with N_R receive antennas simultaneously. Therefore the channel can be represented as a matrix H of size $N_R \times N_T$ and each element h_{ij} of this matrix can be modeled as a complex Gaussian random process. The covariance of the channel is defined as

$$R_0 = E[hh^*], \tag{1}$$

where R_0 is a positive semi-definite (PSD) Hermitian matrix [16], $h = vec(H)$ and $(.)^*$ stands for conjugate

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transpose operation. We use kronecker structure [16] for modeling the channel as below

$$R_0 = R_t^T \otimes R_r, \tag{2}$$

where $R_t = R_t^{\frac{1}{2}} R_t^{\frac{1}{2}H}$ is the transmit antenna correlation and $R_r = R_r^{\frac{1}{2}} R_r^{\frac{1}{2}H}$ is the receive antenna correlation and $\otimes$ stands for kronecker product.

Generally speaking, although there are some models other than kronecker model, we prefer to model the channel covariance with this model. As stated in the kronecker model, transmit and receive correlations are separable. This assumption can be justified by the fact that these arrays are sufficiently far apart with enough random scattering between them. Therefore, the model of the channel can be written as

$$H = R_r^{\frac{1}{2}} H_w R_t^{\frac{1}{2}}, \tag{3}$$

where H_w is an $N_R \times N_T$ i.i.d. complex Gaussian matrix with zero mean and unit variance.

As mentioned in [9], in a practical, common, simplified correlation model, it is assumed that a correlation between the transmit antennas is accrued in a downlink wireless channel where the transmission is done from the base station to the mobile. The received signal at the mobile is a combination of several paths reflected from several local scatters, leading to uncorrelated fading across the receive antennas. Therefore, the channel model reduced to

$$H = H_w R_t^{\frac{1}{2}}. \tag{4}$$

Elements of R_t is illustrated by $R_t = [\rho_{ij}]$, $\forall i, j$ notation and $\rho_{ii} = 1$ where ρ_{ij} is called "correlation coefficient". The system model is illustrated as

$$Y = HX + Z, \tag{5}$$

where Y is an $N_R \times T$ matrix that includes all received signals during T time slots; X is an $N_T \times T$ matrix that includes all transmitted signals; Z is the noise matrix with the variance of σ^2 and H stands for a Rayleigh flat fading channel. The above equation can be re-written as below. In this equation S is the coded stream and F is the precoding matrix which will be discussed later.

$$Y = HFS + Z. \tag{6}$$

The receiver performs maximum-likelihood (ML) detection [17] over a codeword C to obtain

$$\hat{S} = \arg\min_{S \in \mathbb{C}} \|Y - HFS\|_F^2, \tag{7}$$

where $\mathbb{C}$ is the STBC codebook and the subscript F denotes the Frobenius norm. Furthermore, the receiver is assumed to know the channel perfectly, i.e. it knows the channel realization while the transmitter only knows the transmit correlation.

Using PEP criterion for this system, an upper bound is achieved regarding [2]

$$P((s^k(t) \to s^l(t)) \le e^{-d_{\min}^2(t)/2}, \tag{8}$$

$$d_{\min}^2(t) = \frac{1}{\sigma^2} \left\| H_W(t) R_t^{\frac{1}{2}} FE \right\|^2, \tag{9}$$

Where this is the probability of detecting the most probable error detection $s^l(t)$ instead of $s^k(t)$ which is transmitted; E is the code error matrix which is defined as

$$\begin{aligned} &E \doteq \arg\min \det[\tilde{E}\tilde{E}^*] \\ &\textit{subject to} \qquad \tilde{E} = s^k(t) - s^l(t). \end{aligned} \tag{10}$$

The definition of E is very important and here it is based on the worst dominant pair. This assumption works well for orthogonal STBC because the worst case is unique. It should be discussed for other forms of space-time coding.

After some mathematical operations regarding [9], the cost function for finding the desired precoder is

$$\begin{aligned} &\max_F J = \det(I + \frac{1}{\sigma^2} R_t^{\frac{1}{2}} FEE^* F^* R_{a,T}^{*\frac{1}{2}}) \\ &\textit{subject to} \qquad \text{Trace(FF*)}=P_0 . \end{aligned} \tag{11}$$

In above equation, E is the minimum distance code error matrix defined in [9] and P_0 is the total transmitted power. For finding on optimal solution for this cost function, the optimization theory should be used. Finally, solution of the optimization problem is achieved as follows [9]

$$\begin{aligned} &F = V_r \Phi_f V_e^*, \\ &\Phi_f^2 = (\gamma I - \Lambda_r^{-2} \Lambda_e^{-1})_+ \quad . \end{aligned} \tag{12}$$

where $\gamma > 0$ is a positive definite constant calculated from the trance constraint; V_r and Λ_r come from singular value decomposition (SVD) of the correlation matrix R; $(.)_+$ sign means max(.,0) and finally V_e^* and Λ_e^{-1} are calculated from eigenvalue decomposition (EVD) of the distance error matrix E.

$$R_t^{1/2} = U_r \Lambda_r V_r^*, \tag{13}$$

$$EE^* = V_e \Lambda_e V_e^*. \tag{14}$$

In above equations, Λ_r and Λ_e are diagonal eigenvalue matrices and the others are orthogonal eigenmatrices which form a basis for R and E.

III. STBC CDMA SYSTEM IN SPATIALLY CORRELATED CHANNEL

In this paper, we consider the STBC-CDMA downlink system with K users; two transmit antennas and one/two receiver antenna(s).

The transmitted baseband data for every user is $S_{k,i} = (b_k C_k)_i$ which is transmitted through 2 time durations from two antennas as described in Alamouti's structure. b_k is the kth user's data and C_k is its corresponding spreading code. Let $h_{k,j,i}$ be the channel coefficient between ith transmit antenna and the jth receiver antenna of the kth user. The received signal at the jth receiver antenna at the tth and (t+1)th time interval can be written as equations (15) and (16), respectively.

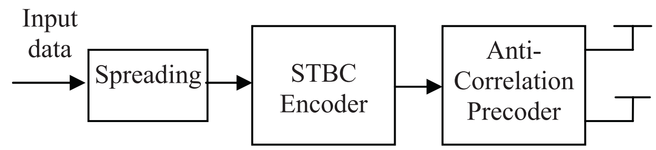
$$r_j^t = \sum_{k=1}^{K} h_{k,j,1} S_{k,1} + \sum_{k=1}^{K} h_{k,j,2} S_{k,2} + n_j^t, \tag{15}$$

$$r_j^{t+1} = \sum_{k=1}^{K} h_{k,j,1}(-S_{k,2}^*) + \sum_{k=1}^{K} h_{k,j,2}(S_{k,1}^*) + n_j^{t+1}. \tag{16}$$

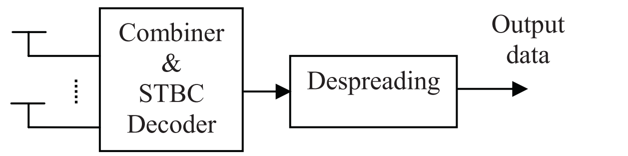
In these equations we assumed that the channel coefficient remains constant over two time periods in which a space-time codeword completes. Moreover, we assume that the channel coefficients are correlated.

As we will show in section IV, this effect reduces the performance of the system. Therefore, in this section we use the precoding technique to mitigate this effect.

The block diagram of the proposed system is shown in figure 1. As shown in this figure, the spread space-time coded data of every user is processed by an anti-correlation precoder matrix, prior to transmission. As all users' data is transmitted from the same antennas, the correlation effect and so the precoder matrix is similar for all users. In order to design the precoder matrix, we construct the matrices for every separate user as follows.



(a) Transmitter



(b) Receive

Figure 1. System block diagram

The data matrix due to kth user is $S_k(t) = \begin{bmatrix} S_{k,1} & -S_{k,2}^* \\ S_{k,2} & S_{k,1}^* \end{bmatrix}$, and the kth user's channel matrix is $H_k(t) = \begin{bmatrix} h_{k,1,1} & h_{k,1,2} \\ h_{k,2,1} & h_{k,2,2} \end{bmatrix}$ for 2*2 system and $H_k(t) = [h_{k,1,1} \quad h_{k,1,2}]$ for 2*1 system. In a correlated environment, $H_k(t) = H_{kw}(t) R_t^{1/2}$. Now we could design the precoder as described in section II. The transmitted data matrix for every user is now $FS_k(t)$.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, some simulation results are provided. All the figures are obtained via simulation process with MATLAB. We assumed the channel is frequency flat fading MIMO wireless channel [17].

Figure 2 shows the BER of a single user STBC system in both uncorrelated and correlated channels. The correlation coefficient is 0.7 for the simulation of this figure and both 2*1 and 2*2 structures are considered. In this figure it is shown that precoder can perfectly mitigate the effect of antenna correlation. To investigate multiuser STBC CDMA system, we consider both 2*1 and 2*2 structures. The spreading factor is considered 16.

It is supposed that base station transmits data for 2 active users in figure 3, the performance of the STBC CDMA system is shown in correlated channel with various correlation coefficients, in comparison with the uncorrelated channel. It is apparent that channel correlation degrades the performance of the CDMA system. This effect is more obvious in 2*2 system and for greater correlation coefficients.

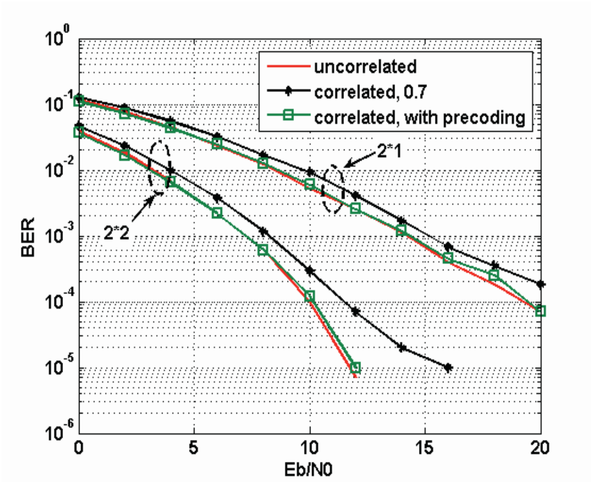


Figure 2. BER of the STBC single user system in correlated channel considering the effect of precoding.

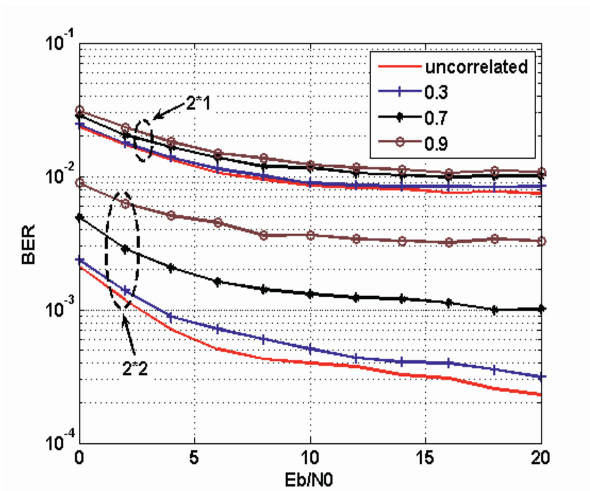


Figure 3. BER of the STBC Multiuser CDMA system in correlated channel considering the effect of correlation coefficient value.

In figure 4 the effect of using anti-correlation precoder is investigated. The correlation coefficient is considered 0.7. It is proved that using this type of precoder could improve the performance of the STBC multiuser CDMA system in correlated channel. In 2*1 system the precoder could almost perfectly mitigates the correlation effect. In 2*2 system the precoder can improve the performance but not completely compensate it. It is also apparent in the figure that the multiuser interference prevents that this effect is compensated completely. However, as it is shown in the figure, for 2*2 scenario, at BER=10^{-3} the correlation effect causes about 15 dB degradation in performance! The precoder can reduce this value to about 3dB!

To consider the multiuser effect more thoroughly, in figure 5 the BER curves are provided against number of interfering users, at $E_b / N_0 = 15dB$. It is shown also in this figure that the procoder could improve the performance of the system in correlation channel. A good example for this situation is shown in 2*2 system. Considering the figure, it is clear that for achieving BER=10^{-3}, 4 other users, besides main user, could be activated in uncorrelated channel, but this value reduces to 2 in correlated channel and precoder increases it to 3. Hence, the BER of the system increases as the number of users grows. However, the multiuser interference has not a destructive effect on the precoder role. On the other hand, it means that we can use anti-correlation precoder to improve the performance of the multiuser system, despite how great the number of users is.

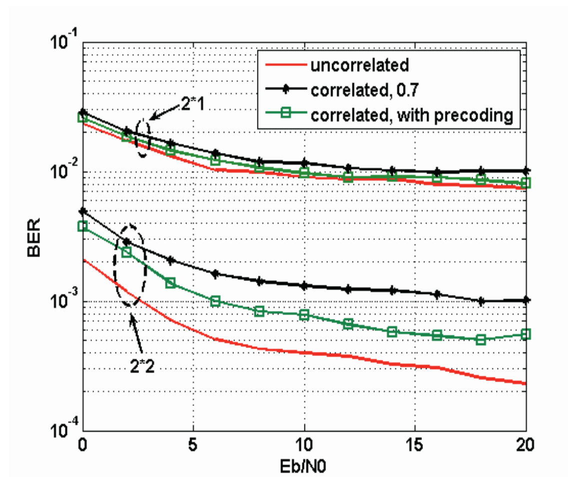


Figure 4. BER of the STBC Multiuser CDMA system in correlated channel considering the effect of precoding.

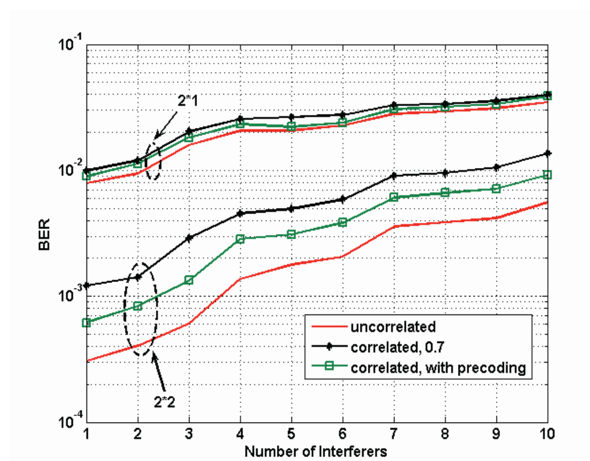


Figure 5. BER of the STBC Multiuser CDMA system in correlated channel considering the effect of number of interfering users.

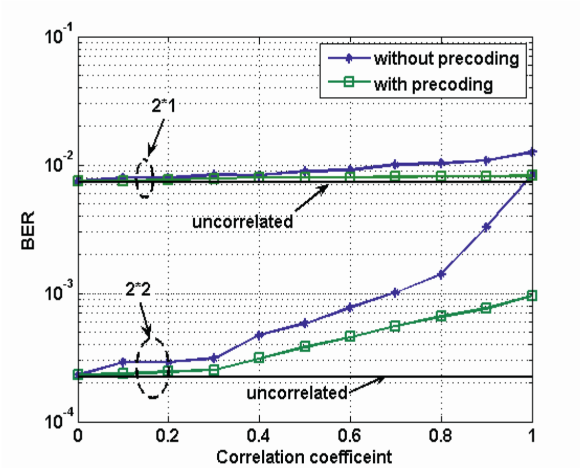


Figure 6. BER of the STBC Multiuser CDMA system in correlated channel, against correlation coefficient value considering the effect of precoding.

In figure 6, the BER curves are given with respect to correlation coefficient. Two active users are considered for this simulation at $E_b / N_0 = 20dB$. In this figure, the performance of the system is shown in correlated channels for conventional system and precoded system. The reference lines due to system BER in uncorrelated channel are also plotted in the figure for better comparison. It is apparent that greater correlation coefficient has worse effect on the performance of the system. However, the precoder could mitigate this effect satisfyingly.

These simulations also demonstrate that the STBC CDMA system in 2*2 scenario is more sensitive to the correlation effect than 2*1 realization.

V. CONCLUSION

In this paper, the effect of spatial correlation in multiuser STBC system was investigated. The system was modeled and simulated in MIMO correlated channel. Simulation results showed that this effect reduces the performance of the STBC CDMA system. However, having some information about the channel of each active user at the transmitter, a linear precoder was exploited to mitigate the effect of correlation in multiuser system. In order to design this precoder, the equations of STBC CDMA system, was reconfigured in this paper. Simulation results illustrated that despite the number of interfering users, the anti-correlation precoder could mitigate the performance degradation caused by correlation. It means that we can use anti-correlation precoder to improve the performance of the multiuser system in correlated MIMO channel.

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