Performance Improvement of STBC CDMA System in Correlated MIMO Channels

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***Abstract*—In this paper the effect of spatial correlation in Code Division Multiple Access (CDMA) Space-Time Block Coding (STBC) system is investigated. Having some information about the channel of each active user in the transmitter, a linear precoder can be exploited in downlink situation to mitigate the effect of correlation in multiuser system environment which has not been addressed before. Simulation results illustrate that despite the number of interfering users, the anti-correlation precoder could mitigate the performance degradation caused by correlation.**

Keywords- STBC; Correlated channel; Precoding; CDMA; multiuser

I. Introduction

Recent researches demonstrate that Multiple-input multiple-output (MIMO) structure can be used in wireless systems to achieve higher performance, and coding gain. In conventional design of Space-time block coding (STBC) [1-3], it is assumed that the code design criteria are calculated due to uncorrelated condition and also there is no knowledge of the channel (CSI) at the transmitter. This may not be accurate in practical situations where multiple antennas are spatially correlated. The effect of spatial correlation on the MIMO channel capacity has been addressed in [4]. However by exploiting the CSI combined with STBC at the transmitter, a precoder can be designed to reduce the problem of spatial correlation [5-6]. Moreover, as stated in [7], if an STBC is capacity lossless for an i.i.d. MIMO channel, then combining this STBC with a linear precoder is capacity optimal for the channel with CSIT [5]. Pairwise error probability (PEP) [7] is regarded in this paper as the performance measurement criterion.

On the other hand, the effect of channel correlation is addressed in the literature for single user scenarios [10-16]. Working with the 3G systems and 4G candidates, the multiple access techniques should be considered.

Motivated by this fact, in this paper we investigate a multiuser STBC CDMA system in correlated MIMO channel. The main contribution of this paper is using an anti-correlation precoder for every separate user to improve the bit error rate (BER) performance of the system in correlated channel. Despite the number of interfering users, this type of precoder could mitigate the performance degradation caused by correlation.

The rest of the paper is organized as follows. Section II describes the anti-correlation precoder for the single user case. In section III STBC CDMA system in spatially correlated channel is considered. The results and comprehensive discussions are presented in section IV. Finally Section V concludes the paper.

II. Anti-Correlation Precoder for STBC Single User

We consider a system in which signals are transmitted via N_T transmit antennas and are received with N_R receive antennas simultaneously. Therefore

$$\mathbf{Y} = \mathbf{HFS} + \mathbf{N}, \tag{1}$$

where $\mathbf{Y}$ is an $N_R \times T$ matrix that includes all received signals during T time slots; $\mathbf{S}$ is an $N_T \times T$ matrix that includes all coded transmitted signals, $\mathbf{H}$ is an $N_R \times N_T$ channel matrix stands for a Rayleigh fading channel, $\mathbf{N}$ is the additive white Gaussian noise (AWGN) and $\mathbf{F}$ is the precoding matrix which will be discussed later.

The receiver performs maximum-likelihood (ML) detection to obtain

$$\hat{\mathbf{S}} = \arg\min_{\mathbf{S}\in\mathbb{C}} \left\|\mathbf{Y} - \mathbf{HFS}\right\|_F^2, \tag{2}$$

where $\mathbb{C}$ is the STBC codebook, and the subscript F denotes the Frobenius norm.

In general form, as stated in the kronecker model [17], transmit and receive correlations are separable. This assumption can be justified by the fact that these arrays are sufficiently far apart with enough random scattering between them. Therefore, the model of the channel can be written as

$$\mathbf{H} = \mathbf{R_r}^{1/2}\mathbf{H_w}\mathbf{R_t}^{1/2}, \tag{3}$$

where $\mathbf{H_w}$ is the i.i.d. complex Gaussian matrix with zero mean and unit variance, $\mathbf{R_t} = \mathbf{R_t}^{1/2}\mathbf{R_t}^{1/2H}$ is the transmit antenna

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correlation and $\mathbf{R_r} = \mathbf{R_r}^{1/2}\mathbf{R_r}^{1/2H}$ is the receive antenna correlation respectively and superscript H denotes the Hermitian operation. Hence, in a practical and simplified correlation model, the channel model reduced to

$$\mathbf{H} = \mathbf{H_w}\mathbf{R_t}^{1/2}. \tag{4}$$

Elements of $\mathbf{R_t}$ are illustrated by $\mathbf{R_t} = [\rho_{ij}]\ , \forall i\,, j$ and $\rho_{ii} = 1$ where ρ_{ij} is called "correlation coefficient".

The receiver is assumed to know the channel perfectly, i.e. it knows the channel realization while the transmitter only knows the transmit correlation. Using PEP criterion for this system, regarding [2]

$$P((s^k(t) \to s^l(t)) \le e^{-d^2_{\min}(t)/2}, \tag{5}$$

$$d^2_{\min}(t) = \left\| \mathbf{H_W(t)R_t^{1/2}FE} \right\|^2. \tag{6}$$

This is the probability of detecting the most probable error detection $s^l(t)$ instead of a different code $s^k(t)$ which is transmitted. After some mathematical operations regarding [9], the cost function of finding the desired precoder is

$$\max_{\mathbf{F}} J = \det(\mathbf{I} + \mathbf{R_t^{1/2}FEE^*F^*R_t^{*1/2}}),$$
$$\text{Subject to } \operatorname{Trace}(\mathbf{FF^*}) = \mathrm{P}_0 = 1, \tag{7}$$

where $\mathbf{E}$ is the minimum distance code error matrix defined in [9]. For finding on optimal solution for this cost function, the optimization theory should be used. Finally, solution of the optimization problem is achieved as follows [9]

$$\mathbf{F} = \mathbf{V_r\Phi_f V_e^*}, \tag{8}$$

$$\mathbf{\Phi^2_f} = (\gamma\mathbf{I} - \mathbf{\Lambda_r^{-2}\Lambda_e^{-1}}), \tag{9}$$

Where γ is a positive constant calculated from the trance constraint, V_r and Λ_r come from singular value decomposition (SVD) of the correlation matrix $\mathbf{R}$, and finally V_e^* and Λ_e^{-1} are calculated from eigenvalue decomposition (EVD) of the distance error matrix E.

III. STBC CDMA SYSTEM IN SPATIALLY CORRELATED CHANNEL

In this paper, we consider the STBC-CDMA downlink system with K users; two transmit antennas and one/two receiver antenna(s). The transmitted baseband data for every user is $S_{k,i} = (b_k C_k)_i$ which is transmitted through 2 time durations from two antennas as described in Alamouti's structure. b_k is the kth user's data and C_k is its corresponding spreading code. Let $h_{k,j,i}$ be the channel coefficient between ith transmit antenna and the jth receiver antenna of the kth user. The received signal at the jth receiver antenna at the tth and (t+1)th time interval can be written as equations (10) and (11), respectively.

$$r_j^t = \sum_{k=1}^{K} h_{k,j,1} S_{k,1} + \sum_{k=1}^{K} h_{k,j,2} S_{k,2} + n_j^t \tag{10}$$

$$r_j^{t+1} = \sum_{k=1}^{K} h_{k,j,1}(-S_{k,2}{}^*) + \sum_{k=1}^{K} h_{k,j,2}(S_{k,1}{}^*) + n_j^{t+1} \tag{11}$$

In these equations we assumed that the channel coefficient remains constant over two time periods in which a space-time codeword completes. Moreover, we assume that the channel coefficients are correlated. As we will show in section IV, this effect reduces the performance of the system. Therefore, in this section we use the precoding technique to mitigate this effect. The block diagram of the proposed system is shown in figure 1. As shown in this figure, the spread space-time coded data of every user is processed by an anti-correlation precoder matrix, prior to transmission. As all users' data is transmitted from the same antennas, the correlation effect and so the precoder matrix is similar for all users. In order to design the precoder matrix, we construct the matrixes for every separate user as follows.

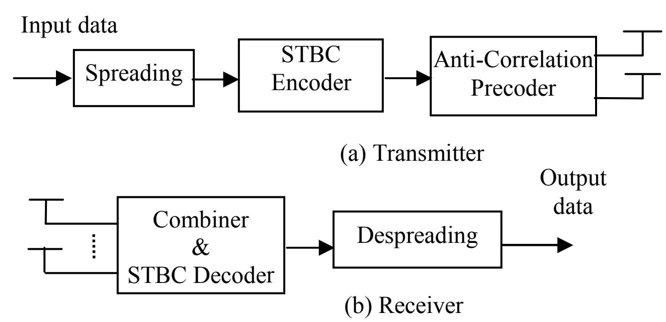

Figure 1. System block diagram

The data matrix due to kth user is $\mathbf{S_k}(t) = \begin{bmatrix} S_{k,1} & -S_{k,2}{}^* \\ S_{k,2} & S_{k,1}{}^* \end{bmatrix}$, and the kth user's channel matrix is $\mathbf{H_k}(t) = \begin{bmatrix} h_{k,1,1} & h_{k,1,2} \\ h_{k,2,1} & h_{k,2,2} \end{bmatrix}$ for 2×2 system and $\mathbf{H_k}(t) = \begin{bmatrix} h_{k,1,1} & h_{k,1,2} \end{bmatrix}$ for 2×1 system. In a correlated environment, $\mathbf{H_k(t)} = \mathbf{H_{kw}(t)R_t^{1/2}}$. Now we could design the precoder as described in section II. The transmitted data matrix for every user is now $\mathbf{FS_k}(t)$.

IV. Results and Discussion

In this section, some simulation results are provided. Figure 2 shows the BER of a single user STBC system in both uncorrelated and correlated channels. The correlation coefficient is 0.7 and both 2×1 and 2×2 structures are considered. It is shown that precoder can perfectly mitigate the effect of antenna correlation. For investigating multiuser STBC CDMA system, both 2×1 and 2×2 structures are considered. The spreading factor is considered 16. It is assumed that base station transmits data for 2 active users. In figure 3, the performance of the STBC CDMA system in correlated channel is shown with various correlation coefficients, in comparison with the uncorrelated channel. It is apparent that channel correlation degrades the performance of the CDMA system. This effect is more obvious in 2×2 system and for greater correlation coefficients. In figure 4 the effect of using anti-correlation precoder is investigated. The correlation coefficient is considered 0.7 in this figure. It is apparent that using this type of precoder could improve the performance of the system in correlated channel. In 2×1 system the precoder could almost perfectly mitigates the correlation effect, however in 2×2 system the precoder can improve the performance but not completely compensate it. It is also apparent in the figure that the multiuser interference prevents that this effect is compensated completely. However, for 2×2 scenario, at BER=10^{-3} the correlation effect causes about 15 dB degradation in performance! The precoder can reduce this value to about 3dB! To consider the multiuser effect more thoroughly, in figures 5, 6 and 7 the BER curves are provided against number of interfering users for 2×2 scenario, at $E_b/N_0=15dB$. It is shown that the procoder could improve the performance of the system in correlated channel. As a good example for this result, for achieving BER=10^{-3}, figure 5 shows that 4 other users besides main user could be activated in uncorrelated channel. This value reduces to 2 in correlated channel, as shown in figure 5 and 6. Precoder increases it to 3 users. Hence, the BER increases as the number of users grows. However, the multiuser interference has not a destructive effect on the precoder role. It means that we can use anti-correlation precoder to improve the performance of the multiuser system, despite how great the number of users are.

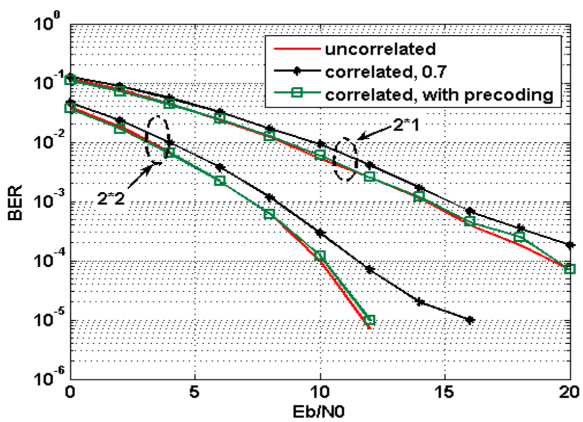

Figure 2. BER of the STBC single user system in correlated channel. Considering the effect of precoding.

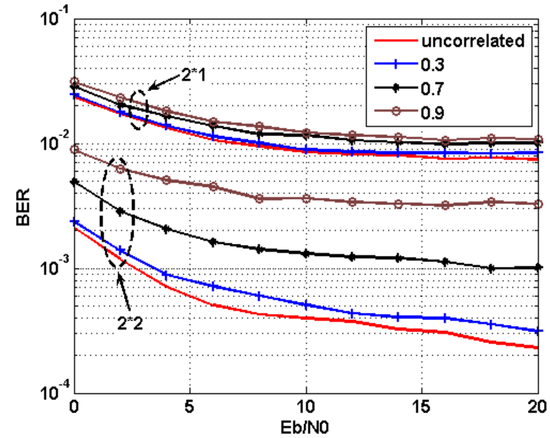

Figure 3. BER of the STBC Multiuser CDMA system in correlated channel. Considering the effect of correlation coefficient value.

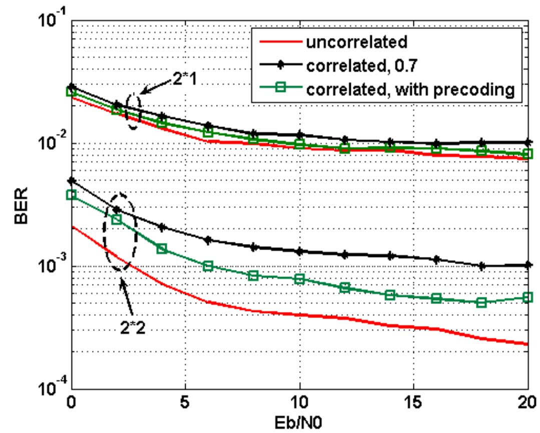

Figure 4. BER of the STBC Multiuser CDMA system in correlated channel. Considering the effect of precoding.

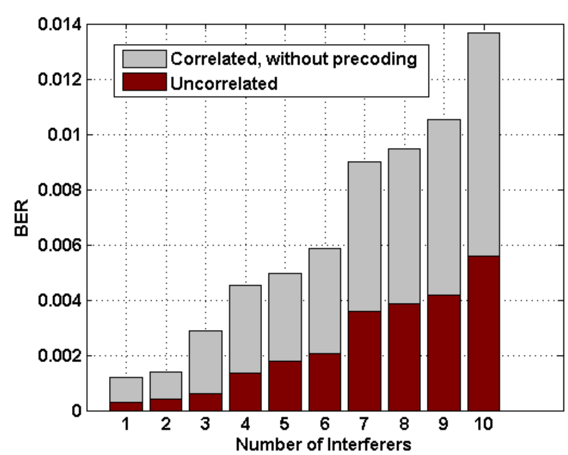

Figure 5. BER of the 2×2 STBC CDMA in correlated channel without precoding in comparison with uncorrelated system considering the effect of increasing the number of interfering users.

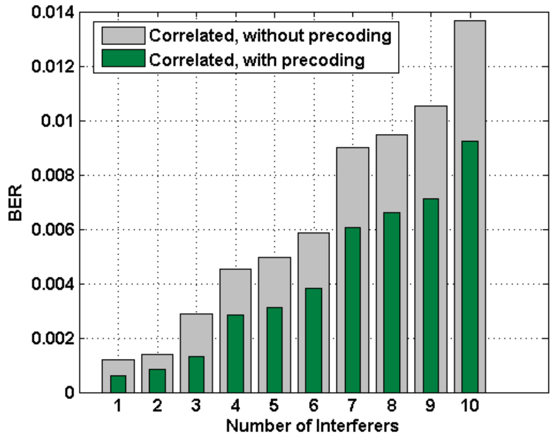

Figure 6. BER of the 2×2 STBC CDMA correlated system with precoding in comparison with the system without precoding considering the effect of increasing the number of interfering users

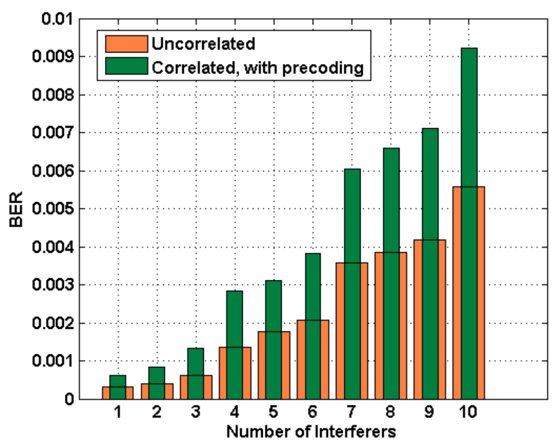

Figure 7. BER of the 2×2 STBC CDMA correlated system with precoding in comparison with the uncorrelated system considering the effect of increasing the number of interfering users

V. CONCLUSION

In this paper, the effect of spatial correlation in multiuser STBC system was investigated. It was shown that this effect reduces the performance of the STBC CDMA system. Having some information about the channel of every active user in the transmitter, a linear precoder was exploited in downlink situation to mitigate the effect of correlation in multiuser system. Simulation results illustrated that despite the number of interfering users, the anti-correlation precoder could mitigate the performance degradation caused by correlation. It means that we can use anti-correlation precoder to improve the performance of the multiuser system in correlated MIMO channel.

REFERENCES

[1] S. Alamouti, “A simple transmitter diversity scheme for wireless communications,” *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[2] V. Tarokh, N. Seshadri, and A.R. Calderbank, “Space-Time Codes for High Data Rate Wireless Communications: performance Criterion and Code Construction,” *IEEE Transactions on Information Theory*, vol. 44, no. 2, pp. 744-765, 1998.

[3] V. Tarokh, H. Jafarkhani, and A.R. Calderbank, “Space-time block codes from orthogonal designs,” *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1456-1467, Jul. 1999.

[4] D. S. Shiu, G.J. Foschini, M.J. Gans, and J.M. Khan, “Fading correlation and its effect on the capacity of multielement antenna systems,” *IEEE Transactions on Com*munications, vol.48, pp.502-513, Mar. 2000.

[5] G. Jongren, M. Skoglund, and B. Ottersten, “Combining beamforming and orthogonal space-time block coding,” *IEEE Transactions on Information Theory*, vol. 48, no. 3, pp. 611–627, Mar. 2002.

[6] S. Zhou and G. B. Giannakis, “Optimal transmitter eigen-beamforming and space-time block coding based on channel correlations,” *IEEE Transactions on Information Theory*, vol. 49, no. 7, pp. 1673–1690, Jul. 2003.

[7] M. Vu and A. Paulraj, “Optimal linear precoders for MIMO wireless correlated channels with nonzero mean in space-time coded systems,” *IEEE Transactions on Signal Processing*, vol. 54, no. 6, pp. 2318–2332, Jun. 2006.

[8] S. Jafar and A. Goldsmith, “Transmitter optimization and optimality of beamforming for multiple antenna systems,” *IEEE Transactions on Wireless Communication*, vol. 3, no. 4, pp. 1165–1175, Jul. 2004.

[9] H. Sampath and A. Paulraj, “Linear precoding for space-time coded systems with known fading correlations,” *IEEE Communications Letters*, vol. 6, no. 6, 239 -241, Jun. 2002.

[10] B. Clerckx, C. Oestges, D. Vanhoenacker-Janvier, and L. Vandendorpe, “Robustness of space-time coding in spatially correlated fast-fading MIMO channels,” *IEEE VTC Conference*, vol. 4, pp. 2413 – 2417, Sep. 2005.

[11] T. A. Lamahewa , R. A. Kennedy, T. D. Abhayapala, and V. K. Nguyen, “Spatial precoder design for space-time coded MIMO systems: based on fixed parameters of MIMO channels,” *Wireless Personal Communications Journal*, Vol 43 , Issue 2, pp. 777 – 799, Oct. 2007.

[12] A. Hjorungnes and D. Gesbert, “Precoding of orthogonal space-time block codes in arbitrarily correlated MIMO channels: Iterative and closed-form solutions,” *IEEE Transactions on Wireless Communication*, vol. 6, no. 3, pp. 1072–1082, Mar. 2007.

[13] K. T. Phan, S. A. Vorobyov, C. Tellambura, “Precoder Design for Space-Time Coded Systems Over Correlated Rayleigh Fading Channels Using Convex Optimization,” *IEEE Transactions on Signal Processing*, vol. 57, no. 2, pp. 814–819, Feb. 2009.

[14] A. Hjørungnes and D. Gesbert, “Precoded Orthogonal Space–Time Block Codes Over Correlated Ricean MIMO Channels,” *IEEE Transactions on Signal Processing*, vol. 55, no. 2, pp. 779–783, Feb. 2007.

[15] A. Medles, A. Alexiou, “New Design for Linear Precoding over STBC in the Presence of Channel Correlation,” *IEEE Transactions on Wireless Communi*cations, vol. 6, no. 4, pp. 1203–1207, Apr. 2007.

[16] S. Pandula and B. P. Paris, “Precoder design via approximate joint diagonalization for ricean correlated MIMO channels in STBC systems,” *42nd Annual Conference on Information Sciences and Systems*, Issue 19-21, pp. 1099 – 1103, Mar. 2008.

[17] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, United Kingdom: Cambridge University Press, May 2003.