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Determinism and Chance

It is widely, though perhaps not universally, thought that objective chance and determinism are incompatible. Here are two philosophers voicing this opinion:

"Today I can see why so many determinists, and even ex-determinists who believe in the deterministic character of classical physics, seriously believe in a subjectivist interpretation of probability: it is in a way the only reasonable possibility which they can accept: for objective physical probabilities are incompatible with determinism; and if classical physics is deterministic, it must be incompatible with an objective interpretation of classical statistical mechanics" Karl Popper, *Quantum Theory and the Schism in Physics.*

"To the question of how chance can be reconciled with determinism....my answer is it can't be done....There is no chance without chance. If our world is deterministic there is no chance in save chances of zero and one. Likewise if our world somehow contains deterministic enclaves, there are no chances in those enclaves" . David Lewis in Postscript to "A Subjectivist's Guide to Objective Chance."

Lewis and Popper - who as we will see have rather different views about the nature of chance- agree that if a theory's fundamental dynamical laws are deterministic then whatever probabilities that theory posits cannot be understood as objective but must be interpreted as *subjective* degrees of belief. Neither gives much in the way of reasons for this claim. Perhaps they think it is obvious; and certainly their view is widespread. In case one is looking for an argument here is one. If the laws are deterministic then the initial conditions of the universe together with the laws entail all facts- at least all facts expressible in the vocabulary of the theory. But if that is so there is no *further* fact for a probability statement to be about. Someone who knew the laws and the complete initial condition- a Laplacian Demon- would be in a position to know everything. So what else can the probabilities posited by such theories be then measures of our ignorance? And since, it is assumed we know the laws that ignorance must be ignorance of the initial conditions. Despite this physicists insist on producing explanations and laws involving probabilities even though the systems to which they apply are considered to be deterministic. The problem is that it is very difficult to see how these probabilities can play the roles that they are called upon to play in predictions, explanations, and laws if the are merely subjective.

The most important deterministic theory that includes probabilities is classical statistical mechanics (CSTM). While there are various ways of understanding CSTM I will rely on a very elegant and clear formulation that has recently been proposed by David Albert (2000).[1]

According to this formulation CSTM consists of three postulates: 1) the fundamental dynamical laws (in the classical case these are the Newtonian laws), 2) a uniform probability distribution- the micro-canonical distribution- over the possible phase points at the origin of the universe, and 3) a statement characterizing the origin of the universe as a special low-entropy condition.[2] A classical mechanical system is a collection of n particles whose state k is specified in term of 6n coordinates (thepositions and momentua of each particle) and a force function F(k) that specifies the forces on each particle when the state is k. As is well known the Newtonian dynamical laws are time reversal symmetric. This means that for any history of the positions of particles compatible with the laws there is a reversed history also compatible with the laws. This gives rise to the puzzle of how they can ground time asymmetric laws especially those characteristic of thermodynamics. Albert shows that it is plausible that from these three postulates all the usual thermodynamic principles (suitably corrected) as well as various other(connected) temporal asymmetries follow.

The probability assumption of CSTM is essential to its predications, explanations and laws. As is well known there are microsates consistent with the fundamental dynamical laws that are compatible with both thermodynamic and anti-thermodyanic behavior. For example, there are microstates that realize a block of ice in warm water which the dynamical laws evolve into states in which the ice has melted and the water is a uniform temperature. But there are other microstates that realize a block of ice in warm water which the laws evolve into states which realize a larger block of ice in warmer water! The usual statistical mechanical explanation of why we every block of ice we encounter behaves in the first way and we never encounter a block of ice that behaves in the second way is that *the probability* of a microstate that evolves in the first way is vastly greater than the probability of a microstate that behaves in the second. The micro-canonical distribution is required to account for all thermodynamic phenomena: the diffusion of gases, the melting of ice in water, and a myriad of other time -asymmetric phenomena. It is needed to account for the nature of records, intervention, and so forth.

Although there is a tradition of interpreting the probabilities in CSTM subjectively- as the degrees of belief a scientist does or should have-it is hard to square a merely subjective interpretation with the way the roles that probabilities play in CSTM; especially the way they enter into explanations and laws. Take, for example, the statistical mechanical explanation of why an ice cube placed in warm water melts in a short amount of time. The explanation proceeds roughly by citing that the initial macro condition of the cup of water + ice is one in which on the micro-canonical probability distribution it is overwhelmingly likely that in a short amount of time the cup+ice cube will be near equilibrium; i.e. the ice cube melted. If the probability appealed to in the explanation is merely a subjective degrees of belief then how can it account for the melting of the ice cube? What could one's ignorance of the initial state of the gas have to do with an explanation of its behavior? A second, and perhaps even more powerful reason to think that the CSTM probabilities are not subjective is that generalizations like "ice cubes in warm water melt", "people grow older", etc. are laws; albeit not fundamental laws. In saying that they are laws I mean that they satsify they usual requirements on laws; they support counterfactuals, are used for reliable predications, and are confirmed by and explain their instances. But these laws cannot follow merely from the fundamental dynamical laws since (in almost all the theories currently under consideration by physicists) are time-reversal symmetric. The explanation within CSTM for the lawfulness of these generalizations invokes the micro-canonical probability distribution. Now it just seems incredible that what endows these generalizations with their lawful status is a subjective probability distribution. Surely what the laws are is not determined by our degrees of belief.

So we have a paradox- *the paradox of deterministic probabilities*- If-as Popper and Lewis claim- objective probabilities cannot co-exist with deterministic dynamical laws then the probabilities that occur in these theories are subjective. But if that is so then-as we have just argued-these probabilities cannot ground the lawfulness of principles that are usually taken to be lawful.

Popper argues that the explanatory successes of statistical mechanics are sufficient to show that the dynamical laws that govern gasses etc. are indeterministic. In fact, he thinks that quantum mechanical indeterministic laws explain statistical mechanics- though he doesn't say how. If Popper is claiming that the phenomena of thermodynamics (e.g. that gasses diffuse, ice-cubes melt, etc.) is sufficient to show that the fundamental dynamical laws are indeterministic then he is not correct. We know that there are initial conditions of the universe that together with deterministic dynamical laws entail the truth (or near truth) of thermodynamic generalizations over the history of the universe until now. The trouble is that they don't entail that these generalizations are *lawful*. The conflict is between statistical mechanics which posits probabilities and the fundamental dynamical laws being deterministic. Popper's point may be only that we don't have to face the paradox of deterministic probabilities as far as CSTM is concerned since we now think that the fundamental laws are quantum mechanical and these are indeterministic. But the situation is not so simple. Some versions of quantum theory -like Bohmian mechanics and many worlds- are thoroughly deterministic; Standard text book quantum theory does include an indeterministic dynamical law- the collapse of the wave function but collapses are too few and far in between to produce statistical mechanics. David Albert has recently suggested that spontaneous localization theories like GRW can do the trick; and if they can then the problem concerning statistical mechanical probabilities is resolved.[3] But even if Albert is correct the issue remains that theories with deterministic dynamics seem to make perfectly good sense even though they posit probabilities along with deterministic dynamics. So I want to see whether we can find a way out of the paradox while maintaining the determinism of the fundamental laws.

I will proceed by looking briefly at the main ideas concerning the nature of chance and see whether any can get us out of the paradox; whether any are both plausible accounts of chance in scientific theories and are compatible with determinism. But first it is important to distinguish our question about the nature of chance from two others. Our question is 1. "What facts about the world make it the case that there are objective probabilities and that they have particular values?" i.e. what are the truth makers of a probability statement like the chance that the ice will melt in the next 5 minute is .99? The other two questions are 2. Assuming that there are chances how do we know that a particular chance distribution is the correct one? 3. What is the explanation, if any, of why a particular chance distribution obtains. It is interesting that even if we don't have an answer to 1 standard methods of statistical inference may provide an answer to 2. But it is hard to see how we can make any progress at all concerning 3 without having a view about what kinds of facts probability statements express.

The main interpretations of Probability:[4]

1. Degrees of Belief
    A) Subjectivism: Defenetti, Savage, Skyrms, Jeffrey.
    B) Objective Bayesianism: Carnap, Jaynes, Rosenkrantz, Goldstein.

2. Physical Probability

    A) Actual Frequentist (Actual Time Averages)
    B) Hypothetical Frequentist (Hypothetical Time Averages)
    C) Propensities
    D) Lewis Best System

All of these "interpretations" of probability are realization of the probability axioms. Of course, satisfying the probability axioms is not sufficient to qualify as probability. The areas of regions on the surface of a table satisfy the axioms of probability but are not probabilities. What more do we want of an account of the probabilities that occur in scientific theories e.g. statistical mechanics, Bohmian mechanics; population genetics? Etc. What further conditions should an interpretation of probability meet? The following is a list of conditions on the nature of chance if it is to play its role in the kinds of theories (CSM and BM) that we have been discussing.

1. Probability is a guide to belief. An adequate account of chance should show why chance (and one's beliefs about chances) ought to guide ones belief.

2. Probabilities are real and objective and not merely useful fictions. What the probabilities and probabilistic laws are is independent of what we think they are so any account has to allow for our being mistaken about or not knowing what the probabilities are.

3. Probabilities and probabilistic laws are explanatory. So, for example, that it is a law that the half-life of some substance is 1 second explains why the substance is almost all gone by the end of an hour.

4. Following the scientific method (including statistical testing, Bayesian inference) yields (if any method can) knowledge or rational belief of probabilities and probabilistic laws.

5. Probabilities can sometimes apply to particular events or propositions. So, for example, quantum mechanics gives us the probability of an electron scattering to a detector in a particular experiment.

6. Frequency connection and tolerance: a) if E is a chance processes that can result in outcome A then the frequency with which A occurs on trials of E is intimately connected-objectively and epistemologically with the chance of E resulting in A however the actual frequency may diverge from the chance.

7. Probability distribution over the initial conditions of the universe makes sense.

8. Probabilities different from 0 and 1 are compatible with determinism.

The last two conditions are specific to making sense of probabilities in determinisitic theories like CSTM.

I want to briefly comment on one of the conditions on the list. The first condition is essential to a measure that satisfies the axioms of probability being a chance. David Lewis (1980) has formulated the way he thought-at first- chance ought guide belief as what he calls the principal principle

PP: C(A/p(A)=x &H&T) =x

C is a credence function and H is "admissible" information. The idea is that we should set our degree of belief in a proposition in accord with what we think the chance of A is (or an average weighted by our degrees of belief in various chances) and that degree of belief should be insensitive to conditionalizing on further "admissible" information. (admissible information includes truths about the past and the laws (including probabilistic laws) The proposition that if the chance of an event is almost 1 then that event is certain is a special case of the PP.) An account of probability should explain why PP (or similar principles) hold. Lewis observes that the PP forges connections between frequencies and chance and between evidence (for example frequency evidence) and our beliefs about chance. Thus the PP shows how scientific method (statistical testing) can provide knowledge of chances. It is very important that any putative account of chance provides an explanation of why chance so understood should guide belief. We immediately dismiss the idea that the area of a region of a table is a chance in part because there is no connection between the area and any beliefs that we should have.[5]

Let's quickly dispense with two interpretations of probability. On propensity accounts chance is a measure of the strength of the tendency of one kind of situation to produce another kind of situation. Many find chances understood as propensities mysteriously metaphysical and especially difficult to see why they should guide beliefs. But whatever the merit of these objections it is clear that propensity chances can't solve the paradox of deterministic probabilities since they are inherently dynamical. If there is a deterministic law that specifies that S produces S* then the chance of S's producing S* must be 1. Popper was one of the early advocates of a propensity theory and no doubt this is part of the reason that he thought that determinists must be subjectivists about scientific probabilities different from 0 and 1.

As we have previously discussed subjectivist accounts of probability - accounts which say that any coherent degrees of belief are rational – cannot explain scientific probabilities. Of what *scientific* importance is it that a particular person- even if that person is Boltzmann or Bohm- assigns certain degrees of belief. The fact that someone has certain degrees of belief cannot account for how scientific probabilities explain or how they are connected to laws. Of course if those degrees of belief are in some sense *correct* then the fact that they are correct is scientifically important. But degrees of belief are correct to the extent that they correspond to scientific probabilities. So the question of what scientific probabilities are remains.

Objective Bayesian or logical accounts of probability appear at first to be more promising. These accounts construes scientific probabilities as degrees of belief that are dictated by logic. In statistical mechanics this approach has been especially associated E.T. Jaynes.[6] The idea is that subject to certain objective constraints- say a macroscopic description of a system and the dynamical laws- certain degrees of belief- whether had by anyone or not- are the ones that logic dictates one should have. The principle of logic is some version of the principle of indifference- if one has no information concerning the possibilities compatible with the constraints then assign them equal probabilities. Given that one is ignorant of the exact micro-state of an ice-cube in warm water assign each state equal probability compatible with ones information including information about the past.(and given that the initial condition of the universe was a low entropy state .....) But there are two big problems with this as an account of scientific probabilities. First, it is difficult to state the principle of indifference in a way that avoids contradiction or arbitrariness. Since the number of micro-states is the size of the continuum there are many ways of assigning equal probability to the possibilities. That one such measure strikes us as simple or natural doesn't mitigate its arbitrariness as a description of nature. But the more compelling problem is that these objective degrees of belief are insensitive to the *actual* micro-state. That is, even if the actual state was a very atypical state the standard probability distribution would be dictated by the indifference principle. Not only does this seem wrong but it is hard to see how these probabilities can be explanatory if they are unrelated to the actual micro-state. For a similar reason its hard to see how they could ground laws since they would entail that even in a world whose whole history consisted in decreasing entropy it is overwhelmingly likely that entropy never decreases.

The usual text-book account of probabilities is that the are frequencies- either actual or hypothetical. According to the actual frequency account the probability of event type E occurring in situation S is the actual frequency of Es occurring on S throughout all history. It is sometimes added that only frequencies of sequences that are "random" count as probabilities. Exactly how the randomness requirement is understood will determine whether or not the account is compatible with the sequence being covered by deterministic laws. But in any case, if the frequencies are those of *actual* sequences then the account isn't suitable for the scientific probabilities we are interested in. One problem is, as Goldstein et. al. observed, it doesn't make sense to talk of the actual frequency with which various initial conditions of the universe are realized. Also, scientific probabilities can diverge- even arbitrarily far- from frequencies. That is, scientific probabilities are "frequency tolerant." For example, the frequency of all quantum mechanical measurements of a certain component of spin (there being very few of these) might substantially differ from the quantum mechanical probability.

According to the hypothetical frequency account probabilities are the frequencies -or rather the limits of the frequencies of sequences-that would be obtained if the experiment were repeated infinitely often. This account is frequency tolerant but encounters a host of new problems. For one, if the probability of E occurring on S is x it is still nomologically possible for an infinite sequence of Ss to yield a different frequency limit.

There are what amounts to versions of the frequency accounts that come up in the statistical mechanical literature that are worth a brief mention.[7] There are certain statistical mechanical systems which are "ergodic"[8]. If a system is ergodic then roughly, this means that for almost all (measure 1) initial conditions the trajectories that the micro states have in phase space (position and momentum) takes it through all possible (compatible with its energy) positions in phase space and in the long run on average the amount of time it spends in regions of phase space that are of interest is the same as the statistical mechanical probability of the system being in that region. This suggests to some that the statistical mechanical probability of the system being in region R should just be interpreted as the average time of its being in R. So, for example if the water in a cup is an ergodic system then on average almost all its time will be spent in a region of phase space in which its temperature is uniform. So the probability of its possessing a uniform temperature is just about 1. The idea is interesting for us since this construal of probability is not only compatible with determinism but depends on the underlying dynamics (in virtue of which the system is ergodic) being deterministic. As an *interpretation* of chances in statistical mechanics it is open to lots of objections both technical and non-technical. First, the interpretation has limited applicability since most systems cannot be shown to be ergodic or are can be shown not to be. But in any case, why would the fact that- to give an analogy- say a ball bouncing back in forth in a box eventually passes through all points and spend 20% of its time on average in the infinite long run in region R give us any reason to think that the probability of finding it R when we open the box is .2? There are two answers to this question that advocates of objective time average interpretation give. One is that it takes awhile to open the box and so one can expect that during that time it will spend .2 of its time in region R. The other is that since all we know about the ball is that it spends .2 of its time in region R we should have degree of belief .2 in finding it there when we open the box. As plausible as this sounds–its not very since it appeals to the principle of indifference which leads to contradictions–it isn't relevant to statistical mechanics since we typically know quite a lot about the region in phase space which a system occupies.

In a very significant paper on probabilities in BM Goldstein et. al. Seem to suggest a new approach to understanding chances in deterministic theories. They first claim that the Bohmian distribution over initial particle positions *cannot* be understood as a probability distribution since, they think, probabilities have to do with repeatable experiments, and it makes no sense to think of the world being repeated over and over again. And in any case, as they emphasize, what we care about is the actual world. But they then say that the initial distribution should be interpreted as a measure of *typicality*. That is, types of initial conditions for which the distribution is high are typical. They say that given the dynamical laws of Bohmian mechanics the quantum equilibrium distribution is *natural*. Although they don't quite say it they seem to mean by this something similar to Jaynes' idea that the dynamical laws and certain natural constraints (in this case the constraint is equivariance[9]) logically entail this typicality measure. It is not clear what "typicality" means or why it is sensible to say that our world is typical though not to say that it is probable. However, they treat typicality like a probability measure in that they think it determine degrees of belief. So on the assumption that our world is governed by Bohmian dynamical laws it is reasonable to believe that it is one in which frequencies match (approximately) quantum mechanical probabilities. They then argue that the quantum equilibrium measure entails that in measurements the frequencies of outcomes are those given by qm. All this (especially the last part) is very interesting but, so far as I can see, it doesn't help with our problem of understanding the nature of chance. First, their notion of "typicality" is unexplained and certainly cannot mean what we mean when we say e.g. chicken paprika is a typical Hungarian dish. Whatever, typicality means it behaves like a probability (it guides belief) and is justified by appeal to logic. So it is open to the same objections we made to Jaynes' account. And the frequencies that it gives rise to cannot be scientific probabilities either for the reasons mentioned in our discussion of frequency theories.

So far as I can see none of the accounts so far canvassed come close to resolving the paradox of deterministic probabilities. But there is an account of the nature of objective chance that I think does. This account is due to David Lewis; the same Lewis who claimed that determinism and chance are incompatible. But, as we will see, he is wrong about that.

Lewis' account of chance is closely connected to his account of laws so I will first explain that. According to Lewis laws are or are expressed by contingent generalizations that are entailed by the theory of the world that best combines simplicity and informativeness. Take all deductive systems whose theorems are true. Some are simpler better systematized than others. Some are stronger, more informative than others. These virtues compete: An uninformative system can be very simple, an unsystematized compendium of miscellaneous information can be very informative. The best system is the one that strikes as good a balance as truth will allow between simplicity and strength. How good a balance that is will depend on how kind nature is. A regularity is a law iff it is a theorem of the best system. (1994a)

Chances come into the picture by allowing some of the axioms of the systems vying for the title "best" to specify probabilities. But that raises the problem of how their informativeness can be measured. It will not do to measure it in terms of the information about probabilities since that would leave probability unanalyzed and so not aid in the Humean program. Lewis suggests that informativeness be measured in terms of "fit.". The idea is that the probability of a world history given a system of axioms measures how well the axioms fit the world. Other things being equal; i.e. simplicity and informativeness the greater the probability of the actual history on the system the better the fit. An enormous gain in simplicity can be achieved by allowing probabilistic axioms in this way. For example, consider a long sequence of the outcomes of flips of a coin. It would be very informative to list them all but very complex. The system that says that they are the outcomes of Bernoulli trials with probability ½ is very simple and fits better than any other simple system.

If a world has a unique Best System (in Lewis' sense) then I will call the contingent generalizations it entails the worlds "L-laws" and the probability statements it entails as specifying its "L-chances." Of course, the important question is whether L-laws and L-chances are or do the scientific work of laws and chances. That is a big question. Here I will just mention that it can be argued that L-chances satisfy the first 6 entries on our list of desiderata for an account of chance.[10] Most importantly, it can be shown that L-chances have a claim on our degrees of belief via the PP (or a slight modification of the PP). Also it is easy to show that given the PP L-chances will entail that the frequencies of independent repeated experiments with identical chances will come close to those given by frequency accounts. But what of our last two desiderata whose satisfaction is required to make sense of chances in deterministic theories? Lewis, you recall, claimed that determinism is incompatible with chances different from 1 and 0. Part of the reason he thought this is that he supposed that chances are always subject to dynamical laws of the form "if the history up until t is h then the chance of e at t' is x". Propensity chances seem to be like that. But there is no reason to restrict Lewis chances in this way. If adding a probability distribution over initial conditions- in our examples over initial conditions of the universe- to other laws greatly increases informativeness (in the form of fit) with little sacrifice in simplicity than the system of those laws together with the probability distribution may well be the Best systematization of the occurrent facts. So L-chances can satisfy desiderata 7 and 8 as well.

Here is how this applies to statistical mechanics. Recall that the fundamental postulates of statistical mechanics are the fundamental dynamical laws (Newtonian in the case of classical statistical mechanics), a postulate that the initial condition was one of low entropy, and the probability distribution at the origin of the universe is the micro-canonical distribution conditional on the low entropy condition. The idea then is that this package is a putative Best System of our world. The contingent generalizations it entails are laws and the chances statements it entails give the chances. That it qualifies as a best system is very plausible. It is simple and enormously informative especially relative to our ways of carving up the world macroscopically. How informative can be seen by the fact that from these postulates the thermodynamic generalizations (or rather very close approximations) follow. By being part of the Best System the probability distribution ears its status as a law and is able to confer lawfulness on those generalizations that it (together with the dynamical laws) entails.

The application to Bohmian mechanics is similar. The quantum equilibrium distribution and the two Bohmian dynamical laws form a putative Best System. It is simple (three laws) and enormously informative- as informative as standard quantum mechanics. Because the quantum equilibrium distribution is a law it is able to confer lawfulness on certain of its consequences including the uncertainty relations etc.

It should be clear now how Lewis' account handles the paradox of deterministic probabilities. On the one hand, a best system probability distribution over the initial conditions is objective since it is part of a simple and informative description of the world. It earns its status as a law (and its capacity to explain and ground laws) by being part of the Best System. On the other hand, the probability distribution is completely compatible with deterministic dynamical laws since it concerns the initial conditions and not dynamical probabilities.

Further, while not an ignorance interpretation of probability Lewis' account does have consequences for what degrees of belief someone who accepts a Best system including a probability distribution should have. Take, for example statistical mechanics. From the Principal Principle someone who accepts the statistical mechanical probability distribution as part of a best system should adopt the micro-canonical distribution -subject to the low entropy condition- as her degrees of belief over the initial conditions at the origin. The interesting consequence is that given the information about macroscopic properties of systems that we normally obtain- e.g. by measuring the temperatures of two bodies in contact- when she conditionalizes on this macroscopic information her new distribution will exclude certain micro-states-those incompatible with this information- but the micro-canonical distribution will persist with respect to the remaining possibilities. So Lewis' account provides kind of fact that, via the PP, supports an objective Bayesian account of the degrees of belief we should have. The corresponding situation for Bohmian mechanics is even more striking. It follows from the analysis of Goldstein et. al. that the quantum equilibrium distribution - understood now as degrees of belief- will persist as one obtain information about the wave function; for example by measurement. Further, the dynamical laws will then maintain ones uncertainly in exactly the way specified by the usual quantum mechanical uncertainty relations. So in the case of Bohmian mechanics- and this is different from statistical mechanics- there is something absolute about the quantum equilibrium distribution. Once one has adopted it as one's degrees of belief it puts a limit on how much knowledge one can obtain! Notice that on Lewis' account we don't have to resort to a "new" notion of typicality (although one could take Lewis' account as an analysis of typicality) different from chance- it is chance.

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Despite all the attractions of Lewisian chances that I have mentioned some will be skeptical. One reason is skepticism that a probability distribution over initial conditions should count as a law at all. That there is a pattern repeated throughout the universe is just the sort of thing that cries out for an explanation rather than explains as laws do. But of course there is no question of causally or lawfully explaining the initial probability distribution. (Though, of course, the exact initial condition and the dynamical laws entail the probability distribution as construed by Lewis.) Perhaps something like this uneasiness is what is at the bottom of our desire to find dynamical laws and dynamical chances. The concern might be relieved somewhat if it can be shown that the probability distributions possess certain features that make it a good fit- perhaps the unique fit- with the dynamical laws. For example, the Bohmian distribution is equivariant and the statistical mechanical distribution supports, more or less, the identity of phase averages with time averages.

The other worry is a more metaphysical one. Lewis' accounts of law and chance satisfy a condition he calls "Humean Supervenience." This means that any two worlds that agree on their occurrent facts also agree on their laws and chances. Laws and chances are part and parcel of the system that best summarizes- best by the lights of the interest of science in simplicity and informativeness- the occurrent facts. There is a quite different way of thinking about laws and chances according to which they *govern* and *generate* occurrent facts. Any account like that will fail to be Humeanly supervenient since different laws and chances can generate the same occurrent facts. For example, the propensity account conceives of chances as involved in the production of events. I don't want to argue the merits of the two approaches here except to say that the idea of a law governing or generating events seems to be a metaphysical vestige of the theological origins of the notion of a law of nature. In any case, if the laws are deterministic then only an account on which the chances supervene on the laws and occurrent facts can accommodate objective chances different from 1 and 0. So even if someone doesn't accept Lewis account of laws and chances in general she might find it to be an adequate account of deterministic chance.

Of the accounts of objective chance that, as far as I know, anyone has thought up Lewis' comes the closest to satisfying the conditions chance we laid out and it does so without weird metaphysics. And, as an added bonus, it provides a solution to the paradox of deterministic probabilities.

Barry Loewer
Jerusalem May 21, 2000

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            %\documentclass[runningheads]{svmult}

%\usepackage{makeidx}    % allows index generation
%\usepackage{graphicx}   % standard LaTeX graphics tool
                        % for including eps-figure files
%\usepackage{subeqnar}   % subnumbers individual equations
                        % within an array
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%\usepackage{cropmark}  % cropmarks for pages without
                        % pagenumbers - only needed when manuscript
                        % is printed from paper and not from data
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                        % lecture notes, and the like.
%\makeindex              % used for the subject index
                        % please use the style sprmidx.sty with
                        % your makeindex program

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\toctitle{Determinism and Chance}
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\titlerunning{Determinism and Chance}
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\section*{}

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Lewis and Popper -- who as we will see have rather different views about the
nature of chance -- agree that if a theory's fundamental dynamical laws are
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as objective but must be interpreted as subjective degrees of belief. Neither gives
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ignorance of the initial conditions. Despite this physicists insist on producing
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they apply are considered to be deterministic. The problem is that it is very

difficult to see how these probabilities can play the roles that they are called upon to play in predictions, explanations, and laws if the are merely subjective.

Here I will discuss two deterministic theories that involve probabilities; classical statistical mechanics (CSM) and non-relativistic Bohmian Mechanics (BM). Both theories include deterministic dynamical laws that are complete with respect to the fundamental states of the theories. By this I mean that for an isolated system the state at one time and the laws are sufficient to determine the state at any other time. Both theories also posit probability distributions over initial states that are best understood as initial conditions of the universe (or a state at a time close to the origin of the universe).

While there are various ways of understanding CSTM I will rely on a very elegant and clear formulation that has recently been proposed by David Albert (2000). According to this formulation CSTM consists of three postulates: 1) the fundamental dynamical laws (in the classical case these are the Newtonian laws), 2) a uniform probability distribution -- the micro-canonical distribution -- over the possible phase points at the origin of the universe, and 3) a statement characterizing the origin of the universe as a special low-entropy condition. As is well known the dynamical laws are time reversal symmetric and that gives rise to the puzzle of how they can ground time asymmetric laws especially those characteristic of thermodynamics. Albert shows how from these three postulates thermodynamic principles (suitably corrected) as well as various temporal asymmetries follow.

Non-relativistic Bohmian mechanics is a so-called ``hidden variable'' interpretation of the quantum formalism. Its ontology consists of the quantum wave function $\psi$ construed as a real field (in configuration space) and particles (the ``hidden variables'') which always possess determinate positions X. $\psi(t)$ evolves in accordance with Schr\"odinger's law and $\psi(t)$ also determines the evolution of the particle configuration. Both of the dynamical laws are deterministic. In Bohmian mechanics there is no collapse of the wave function and no measurement problem. Pointers, cats, and so on always possess determinate properties in so far as those properties are determined by the positions of their constituent particles. In addition to the two dynamical laws Bohmian mechanics posits a probability distribution -- the quantum equilibrium distribution $|\psi|^2$ over the possible initial configuration of particle positions. It has been shown that this theories recovers all the standard predictions of text-book quantum mechanics in so far as these are unambiguous and in so far as the outcomes of measurements are recorded in positions.

In both CSTM and BM the probability assumptions are essential to the predications, explanations and laws of their respective theories. In the case of statistical mechanics the micro-canonical distribution is required to account for all thermodynamic phenomena: the diffusion of gases, the melting of ice in water, and a myriad of other time-asymmetric phenomena. It is needed to account for the nature of records, intervention, and so forth. In the case of BM the quantum equilibrium distribution is required to account for the uncertainty relations, the prohibition of super luminal signaling, and indeed all the statistical consequences of quantum mechanics. The two distributions have four features in common that are important to our discussion: a) they co-exist with deterministic dynamical laws b) they are over initial conditions for the entire universe, c) they are thought to be in some sense simple or natural, d) they are absolutely essential to the theories that posit them.

Although there is a tradition of interpreting the probabilities in both theories subjectively -- as the degrees of belief a scientist does or should have -- it is hard to square a merely subjective interpretation with the way the roles that probabilities play in these theories; especially the way they enter into explanations and laws. Take, for example, the statistical mechanical explanation of why an ice cube placed in warm water melts in a short amount of time. The explanation proceeds roughly by citing that the initial macro condition of the cup of water + ice is one in which on the micro-canonical probability distribution it is overwhelmingly likely that in a short amount of time the cup + ice cube will be near equilibrium; i.e.\ the ice cube melted. If the probability appealed to in the explanation is merely a subjective degrees of belief then how can it account for the melting of the ice cube? What could one's ignorance of the initial state of the gas have to do with an explanation of its behavior? Similarly the Bohmian mechanical explanation of e.g.\ diffraction patterns in the two slit experiment essential refers to the quantum equilibrium distribution. But how can the diffraction pattern -- which has nothing to do with our beliefs -- be explained by subjective degrees of belief. A second, and perhaps even more powerful reason to think that the probabilities referred to in these theories are not subjective is that that generalizations like ``ice cubes on warm water melt'', ``people grow older'', etc.\ are laws; albeit not fundamental laws. In saying that they are laws I mean just that they support counterfactuals and are confirmed by and explain their instances. But these laws cannot follow merely from the fundamental dynamical laws since (in almost all the theories currently under consideration by physicists) are time-reversal symmetric. The explanation within CSTM for the lawfulness of these generalizations invokes the micro-canonical probability distribution. The situation in Bohmian Mechanics is even more striking. The uncertainty relations, principles of chemical combination, the impossibility of super-luminal signaling -- which seems as law-like as anything -- within BM all depend on the quantum equilibrium assumption. Now it just seems incredible that what endows these generalizations with their lawful status is a subjective probability distribution. Surely what the laws are is not determined by our degrees of belief.

So we have a paradox -- the paradox of deterministic probabilities: If -- as Popper and Lewis claim -- objective probabilities cannot co-exist with deterministic dynamical laws then the probabilities that occur in these theories are subjective. But if that is so then -- as we have just argued -- these probabilities cannot ground the lawfulness of principles that are usually taken to be lawful.

Popper argues that the explanatory successes of statistical mechanics are sufficient to show that the dynamical laws that govern gasses etc.\ are indeterministic. In fact, he thinks that quantum mechanical indeterministic laws explain statistical mechanics -- though he doesn't say how. If Popper is claiming that the phenomena of thermodynamics (e.g.\ that gasses diffuse, ice-cubes melt, etc.) is sufficient to show that the fundamental dynamical laws are indeterministic then he is not correct. We know that there are initial conditions of the universe that together with deterministic dynamical laws entail the truth (or near truth) of thermodynamic generalizations over the history of the universe until now. The trouble is that they don't entail that these generalizations are lawful. The conflict is between statistical mechanics which posits probabilities and the fundamental dynamical laws being deterministic. Popper's point may be only that we don't have to face the paradox of deterministic probabilities as far as CSTM is concerned since we now think that the fundamental laws are quantum mechanical and these are indeterministic. But the situation is not so simple. Some versions of quantum theory -- like Bohmian mechanics and many worlds -- are thoroughly deterministic; Standard text book quantum theory does include an indeterministic dynamical law -- the collapse of the wave function, but collapses are too few and far in between to produce statistical mechanics. David Albert has recently suggested that spontaneous localization theories like GRW can do the trick; and if they can then the problem concerning statistical mechanical probabilities is resolved. But even if Albert is correct the issue remains that theories like Bohm's and CSTM mechanics with deterministic dynamics seem to make perfectly good sense even though they posit probabilities along with deterministic dynamics. So I want to see whether we can find a way out of the paradox while maintaining the determinism of the fundamental laws.

I will proceed by looking briefly at the main ideas concerning the nature of chance and see whether any can get us out of the paradox; whether any are both plausible accounts of chance in scientific theories and are compatible with determinism. But first it is important to distinguish our question about the nature of chance from two others. Our question is 1. What facts about the world make it the case that there are objective probabilities and that they have particular values? i.e.\ what are the truth makers of a probability statement like ``the chance that the ice will melt in the next 5 minute is .99''? The other two questions are 2. Assuming that there are chances how do we know that a particular chance distribution is the correct one? 3. What is the status of the chance distribution and what explains why it obtains? It is interesting that even if we don't have an answer to 1 standard methods of statistical inference may provide an answer to 2. But it is hard to see how we can make any progress at all concerning 3 without having a view about what kinds of facts probability statements express.

\noindent The main interpretations of Probability:
\begin{enumerate}
  \item Degrees of Belief
  \begin{enumerate}
    \item Subjectivism: Defenetti, Savage, Skyrms, Jeffrey.
    \item Objective Bayesianism: Carnap, Jaynes, Rosenkrantz, Goldstein.
  \end{enumerate}
  \item Physical Probability
  \begin{enumerate}
    \item Actual Frequentist (Actual Time Averages)
    \item Hypothetical Frequentist (Hypothetical Time Averages)
    \item Propensities
    \item Lewis Best System
  \end{enumerate}
\end{enumerate}

All of these ``interpretations'' of probability are realization of the probability axioms. Of course, satisfying the probability axioms is not sufficient to qualify as probability. The areas of regions on the surface of a table satisfy the axioms of probability but are not probabilities. What more do we want of an account of the probabilities that occur in scientific theories e.g.\ statistical mechanics, Bohmian mechanics; population genetics? Etc. What further conditions should an interpretation of probability meet? The following is a list of conditions on the nature of chance if it is to play its role in the kinds of theories (CSM and BM) that we have been discussing.

\begin{enumerate}

\item Probability is a guide to belief. An adequate account of chance should show why chance (and one's beliefs about chances) ought to guide ones belief.

\item Probabilities are real and objective and not merely useful fictions. What the probabilities and probabilistic laws are is independent of what we think they are so any account has to allow for our being mistaken about or not knowing what the probabilities are.

\item Probabilities and probabilistic laws are explanatory. So, for example, that it is a law that the half-life of some substance is 1 second explains why the substance is almost all gone by the end of an hour.

\item Following the scientific method (including statistical testing, Bayesian inference) yields (if any method can) knowledge or rational belief of probabilities and probabilistic laws.

\item Probabilities can sometimes apply to particular events or propositions. So, for example, quantum mechanics gives us the probability of an electron scattering to a detector in a particular experiment.

\item Frequency connection and tolerance: a) if E is a chance processes that can result in outcome A then the frequency with which A occurs on trials of E is intimately connected-objectively and epistemologically with the chance of E resulting in A however the actual frequency may diverge from the chance.

\item Probability distribution over the initial conditions of the universe makes sense.

\item Probabilities different from 0 and 1 are compatible with determinism.

\end{enumerate}

The last two conditions are specific to making sense of probabilities in CSTM and BM since, as we noted, both theories contain deterministic dynamical laws and postulates specifying probability distributions over the initial conditions of the universe.

First though I want to briefly comment on one of the conditions on the list. The first condition is essential to a measure that satisfies the axioms of probability being a chance. David Lewis (1980) has formulated the way he thought-at first- chance ought guide belief as what he calls the principal principle

$$
PP:\quad C(A|p(A)=x \&H\&T) =x
$$

C is a credence function and H is ``admissible'' information. The idea is that we should set our degree of belief in a proposition in accord with what we think the chance of $A$ is (or an average weighted by our degrees of belief in various chances) and that degree of belief should be insensitive to conditionalizing on further ``admissible'' information. (admissible information includes truths about the past and the laws (including probabilistic laws) The proposition that if the chance of an event is almost 1 then that event is certain is a special case of the PP.)
An account of probability should explain why PP (or similar principles) hold. Lewis observes that the PP forges connections between frequencies and chance and between evidence (for example frequency evidence) and our beliefs about chance. Thus the PP shows how scientific method (statistical testing) can provide knowledge of chances. It is very important that any putative account of chance provides an explanation of why chance so understood should guide belief. We immediately dismiss the idea that the area of a region of a table is a chance in part because there is no connection between the area and any beliefs that we should have.

Let's quickly dispense with two of the accounts in our list. On propensity accounts chance is a measure of the strength of the tendency of one kind of situation to produce another kind of situation. Many find chances understood as propensities mysteriously metaphysical and especially difficult to see why they should guide beliefs. But whatever the merit of these objections it is clear that propensity chances can't solve the paradox of deterministic probabilities since they are inherently dynamical. If there is a deterministic law that specifies that S produces S^* then the chance of S's producing S^* must be 1. Popper was one of the early advocates of a propensity theory and no doubt this is part of the reason that he thought that determinists must be subjectivists about scientific probabilities different from 0 and 1.

As we have previously discussed subjectivist accounts of probability -- accounts which say that any coherent degrees of belief are rational -- cannot explain scientific probabilities. Of what scientific importance is it that a particular person -- even if that person is Boltzmann or Bohm -- assigns certain degrees of belief. The fact that someone has certain degrees of belief cannot account for how scientific probabilities explain or how they are connected to laws. Of course if those degrees of belief are in some sense correct then the fact that they are correct is scientifically important. But degrees of belief are correct to the extent that they correspond to scientific probabilities. So the question of what scientific probabilities are remains.

Objective Bayesian or logical accounts of probability appear at first to be more promising. These accounts construes scientific probabilities as degrees of belief that are dictated by logic. In statistical mechanics this approach has been especially associated E.T.~Jaynes. The idea is that subject to certain objective constraints -- say a macroscopic description of a system and the dynamical laws -- certain degrees of belief -- whether had by anyone or not -- are the ones that logic dictates one should have. The principle of logic is some version of the principle of indifference -- if one has no information concerning the possibilities compatible with the constraints then assign them equal probabilities. Given that one is ignorant of the exact micro-state of an ice-cube in warm water assign each state equal probability compatible with ones information including information about the past.(and given that the initial condition of the universe was a low entropy state \ldots) But there are two big problems with this as an account of scientific probabilities. First, it is difficult to state the principle of indifference in a way that avoids contradiction or arbitrariness. Since the number of micro-states is the size of the continuum there are many ways of assigning equal probability to the possibilities. That one such measure strikes us as simple or natural doesn't mitigate its arbitrariness as a description of nature. But the more compelling problem is that these objective degrees of belief are insensitive to the actual micro-state. That is, even if the actual state was a very atypical state the standard probability distribution would be dictated by the indifference principle. Not only does this seem wrong but it is hard to see how these probabilities can be explanatory if they are unrelated to the actual micro-state. For a similar reason its hard to see how they could ground laws since they would entail that even in a world whose whole history consisted in decreasing entropy it is overwhelmingly likely that entropy never decreases.

The usual text-book account of probabilities is that they are frequencies -- either actual or hypothetical. According to the actual frequency account the probability of event type E occurring in situation S is the actual frequency of Es occurring on S throughout all history. It is sometimes added that only frequencies of sequences that are ``random'' count as probabilities. Exactly how the randomness requirement is understood will determine whether or not the account is compatible with the sequence being covered by deterministic laws. But in any case, if the frequencies are those of actual sequences then the account isn't suitable for the scientific probabilities we are interested in. One problem is, as Goldstein et. al. observed, it doesn't make sense to talk of the actual frequency with which various initial conditions of the universe are realized. Also, scientific probabilities can diverge -- even arbitrarily far -- from frequencies. That is, scientific probabilities are ``frequency tolerant.'' For example, the frequency of all quantum mechanical measurements of a certain component of spin (there being very few of these) might substantially differ from the quantum mechanical probability.

According to the hypothetical frequency account probabilities are the frequencies -- or rather the limits of the frequencies of sequences -- that would be obtained if the experiment were repeated infinitely often. This account is frequency tolerant but encounters a host of new problems. For one, if the probability of E occurring on S is x it is still nomologically possible for an infinite sequence of Ss to yield a different frequency limit.

There are what amounts to versions of the frequency accounts that come up in the statistical mechanical literature that are worth a brief mention.5 There are certain statistical mechanical systems which are ``ergodic''. If a system is ergodic then roughly, this means that for almost all (measure 1) initial conditions the trajectories that the micro states have in phase space (position and momentum) takes it through all possible (compatible with its energy) positions in phase space and in the long run on average the amount of time it spends in regions of phase space that are of interest is the same as the statistical mechanical probability of the system being in that region. This suggests to some that the statistical mechanical probability of the system being in region R should just be interpreted as the average time of its being in R. So, for example if the water in a cup is an ergodic system then on average almost all its time will be spent in a region of phase space in which its temperature is uniform. So the probability of its possessing a uniform temperature is just about 1. The idea is interesting for us since this construal of probability is not only compatible with determinism but depends on the underlying dynamics (in virtue of which the system is ergodic) being deterministic. As an interpretation of chances in statistical mechanics it is open to lots of objections both technical and non-technical. First, the interpretation has limited applicability since most systems cannot be shown to be ergodic or are can be shown not to be. But in any case, why would the fact that- to give an analogy- say a ball bouncing back in forth in a box eventually passes through all points and spend 20\% of its time on average in the infinite long run in region R give us any reason to think that the probability of finding it R when we open the box is .2? There are two answers to this question that advocates of objective time average interpretation give. One is that it takes awhile to open the box and so one can expect that during that time it will spend .2 of its time in region R. The other is that since all we know about the ball is that it spends .2 of its time in region R we should have degree of belief .2 in finding it there when we open the box. As plausible as this sounds (its not very since it appeals to the principle of indifference which leads to contradictions) it isn't relevant to statistical mechanics since we typically know quite a lot about the region in phase space which a system occupies.

In a very significant paper on probabilities in BM Goldstein et. al. Seem to suggest a new approach to understanding chances in deterministic theories. They first claim that the Bohmian distribution over initial particle positions cannot be understood as a probability distribution since, they think, probabilities have to do with repeatable experiments, and it makes no sense to think of the world being repeated over and over again. And in any case, as they emphasize, what we care about is the actual world. But they then say that the initial distribution should be interpreted as a measure of typicality. That is, types of initial conditions for which the distribution is high are typical. They say that given the dynamical laws of Bohmian mechanics the quantum equilibrium distribution is natural. Although they don't quite say it they seem to mean by this something similar to Jaynes' idea that the dynamical laws and certain natural constraints (in this case the constraint is equivariance6) logically entail this typicality measure. It is not clear what ``typicality'' means or why it is sensible to say that our world is typical though not to say that it is probable. However, they treat typicality like a probability measure in that they think it determine degrees of belief. So on the assumption that our world is governed by Bohmian dynamical laws it is reasonable to believe that it is one in which frequencies match (approximately) quantum mechanical probabilities. They then argue that the quantum equilibrium measure entails that in measurements the frequencies of outcomes are those given by qm. All this (especially the last part) is very interesting but, so far as I can see, it doesn't help with our problem of understanding the nature of chance. First, their notion of ``typicality'' is unexplained and certainly cannot mean what we mean when we say e.g.\ chicken paprika is a typical Hungarian dish. Whatever, typicality means it behaves like a probability (it guides belief) and is justified by appeal to logic. So it is open to the same objections we made to Jaynes' account. And the frequencies that it gives rise to cannot be scientific probabilities either for the reasons mentioned in our discussion of frequency theories.

So far as I can see none of the accounts so far canvassed come close to resolving the paradox of deterministic probabilities. But there is an account of the nature of objective chance that I think does. This account is due to David Lewis; the same Lewis who claimed that determinism and chance are incompatible. But, as we will see, he is wrong about that.

Lewis' account of chance is closely connected to his account of laws so I will first explain that. According to Lewis laws are or are expressed by contingent generalizations that are entailed by the theory of the world that best combines simplicity and informativeness. Take all deductive systems whose theorems are true. Some are simpler better systematized than others. Some are stronger, more informative than others. These virtues compete: An uninformative system can be very simple, an unsystematized compendium of miscellaneous information can be very informative. The best system is the one that strikes as good a balance as truth will allow between simplicity and strength. How good a balance that is will depend on how kind nature is. A regularity is a law iff it is a theorem of the best system. (1994a)

Chances come into the picture by allowing some of the axioms of the systems vying for the title ``best'' to specify probabilities. But that raises the problem of how their informativeness can be measured. It will not do to measure it in terms of the information about probabilities since that would leave probability unanalyzed and so not aid in the Humean program. Lewis suggests that informativeness be measured in terms of ``fit''. The idea is that the probability of a world history given a system of axioms measures how well the axioms fit the world. Other things being equal; i.e.\ simplicity and informativeness the greater the probability of the actual history on the system the better the fit. An enormous gain in simplicity can be achieved by allowing probabilistic axioms in this way. For example, consider a long sequence of the outcomes of flips of a coin. It would be very informative to list them all but very complex. The system that says that they are the outcomes of Bernoulli trials with probability $1/2$ is very simple and fits better than any other simple system.

If a world has a unique Best System (in Lewis' sense) then I will call the contingent generalizations it entails the worlds ``L-laws'' and the probability statements it entails as specifying its ``L-chances.'' Of course, the important question is whether L-laws and L-chances are or do the scientific work of laws and chances. That is a big question. Here I will just mention that it can be argued that L-chances satisfy the first 6 entries on our list of desiderata for an account of chance. Most importantly, it can be shown that L-chances have a claim on our degrees of belief via the PP (or a slight modification of the PP). Also it is easy to show that given the PP L-chances will entail that the frequencies of independent repeated experiments with identical chances will come close to those given by frequency accounts. But what of our last two desiderata whose satisfaction is required to make sense of chances in deterministic theories? Lewis, you recall, claimed that determinism is incompatible with chances different from 1 and 0. Part of the reason he thought this is that he supposed that chances are always subject to dynamical laws of the form ``if the history up until t is h then the chance of e at t' is x''. Propensity chances seem to be like that. But there is no reason to restrict Lewis chances in this way. If adding a probability distribution over initial conditions- in our examples over initial conditions of the universe- to other laws greatly increases informativeness (in the form of fit) with little sacrifice in simplicity than the system of those laws together with the probability distribution may well be the Best systematization of the occurrent facts. So L-chances can satisfy desiderata 7 and 8 as well.

Here is how this applies to statistical mechanics. Recall that the fundamental postulates of statistical mechanics are the fundamental dynamical laws (Newtonian in the case of classical statistical mechanics), a postulate that the initial condition was one of low entropy, and the probability distribution at the origin of the universe is the micro-canonical distribution conditional on the low entropy condition. The idea then is that this package is a putative Best System of our world. The contingent generalizations it entails are laws and the chances statements it entails give the chances. That it qualifies as a best system is very plausible. It is simple and enormously informative especially relative to our ways of carving up the world macroscopically. How informative can be seen by the fact that from these postulates the thermodynamic generalizations (or rather very close approximations) follow. By being part of the Best System the probability distribution ears its status as a law and is able to confer lawfulness on those generalizations that it (together with the dynamical laws) entails.

The application to Bohmian mechanics is similar. The quantum equilibrium distribution and the two Bohmian dynamical laws form a putative Best System. It is simple (three laws) and enormously informative- as informative as standard quantum mechanics. Because the quantum equilibrium distribution is a law it is able to confer lawfulness on certain of its consequences including the uncertainty relations etc.

It should be clear now how Lewis' account handles the paradox of deterministic probabilities. On the one hand, a best system probability distribution over the initial conditions is objective since it is part of a simple and informative description of the world. It earns its status as a law (and its capacity to explain and ground laws) by being part of the Best System. On the other hand, the probability distribution is completely compatible with deterministic dynamical laws since it concerns the initial conditions and not dynamical probabilities.

Further, while not an ignorance interpretation of probability Lewis' account does have consequences for what degrees of belief someone who accepts a Best system including a probability distribution should have. Take, for example statistical mechanics. From the Principal Principle someone who accepts the statistical mechanical probability distribution as part of a best system should adopt the micro-canonical distribution -- subject to the low entropy condition -- as her degrees of belief over the initial conditions at the origin. The interesting consequence is that given the information about macroscopic properties of systems that we normally obtain -- e.g.\ by measuring the temperatures of two bodies in contact --

when she conditionalizes on this macroscopic information her new distribution will exclude certain micro-states -- those incompatible with this information -- but the micro-canonical distribution will persist with respect to the remaining possibilities. So Lewis' account provides kind of fact that, via the PP, supports an objective Bayesian account of the degrees of belief we should have. The corresponding situation for Bohmian mechanics is even more striking. It follows from the analysis of Goldstein et. al. that the quantum equilibrium distribution -- understood now as degrees of belief -- will persist as one obtain information about the wave function; for example by measurement. Further, the dynamical laws will then maintain ones uncertainly in exactly the way specified by the usual quantum mechanical uncertainty relations. So in the case of Bohmian mechanics -- and this is different from statistical mechanics -- there is something absolute about the quantum equilibrium distribution. Once one has adopted it as one's degrees of belief it puts a limit on how much knowledge one can obtain! Notice that on Lewis' account we don't have to resort to a ``new'' notion of typicality (although one could take Lewis' account as an analysis of typicality) different from chance -- it is chance.

Despite all the attractions of Lewisian chances that I have mentioned some will be skeptical. One reason is skepticism that a probability distribution over initial conditions should count as a law at all. That there is a pattern repeated throughout the universe is just the sort of thing that cries out for an explanation rather than explains as laws do. But of course there is no question of causally or lawfully explaining the initial probability distribution. (Though, of course, the exact initial condition and the dynamical laws entail the probability distribution as construed by Lewis.) Perhaps something like this uneasiness is what is at the bottom of our desire to find dynamical laws and dynamical chances. The concern might be relieved somewhat if it can be shown that the probability distributions possess certain features that make it a good fit -- perhaps the unique fit -- with the dynamical laws. For example, the Bohmian distribution is equivariant and the statistical mechanical distribution supports, more or less, the identity of phase averages with time averages.

The other worry is a more metaphysical one. Lewis' accounts of law and chance satisfy a condition he calls ``Humean Supervenience.'' This means that any two worlds that agree on their occurrent facts also agree on their laws and chances. Laws and chances are part and parcel of the system that best summarizes -- best by the lights of the interest of science in simplicity and informativeness -- the occurrent facts. There is a quite different way of thinking about laws and chances according to which they govern and generate occurrent facts. Any account like that will fail to be Humeanly supervenient since different laws and chances can generate the same occurrent facts. For example, the propensity account conceives of chances as involved in the production of events. I don't want to argue the merits of the two approaches here except to say that the idea of a law governing or generating events seems to be a metaphysical vestige of the theological origins of the notion of a law of nature. In any case, if the laws are deterministic then only an account on which the chances supervene on the laws and occurrent facts can accommodate objective chances different from 1 and 0. So even if someone doesn't accept Lewis account of laws and chances in general she might find it to be an adequate account of deterministic chance.

Of the accounts of objective chance that, as far as I know, anyone has thought up Lewis' comes the closest to satisfying the conditions chance we laid out and it does so without weird metaphysics. And, as an added bonus, it provides a solution to the paradox of deterministic probabilities.

\footnote{While the basic outlines of Albert's approach are not unfamiliar in the statistical Mechanical literature (something like this can be read into Boltzmann) it must be admitted that the foundations of statistical mechanics are as controversial as the foundations of quantum mechanics. For a magisterial survey of current work in the foundations of statistical mechanics see \cite{Sklar (1994)}.}

\footnote{The relevant notion of entropy is that characterized by Boltzamann. According to it entropy applies to a micro-state and is given by the size (under the probability assumption) of the macro-conditions that the micro-state realizes.}

\footnote{Expositions of Bohm's theory can be found in \cite{Bohm,Bohm and Hiley (1993),Bell,Albert (1992),Durr et al}}

\footnote{See \cite{Albert (1994)}. Albert shows how the statistical mechanical probabilities can be reduced to those of GRW.
Accounts of interpretation of probability can be found in \cite{Sklar (1994),Howson and Urbach}.
Of course, if we think that the chance that a coin is located in a region is proportional to the area of the region then there will be a connection between areas and degrees of belief -- but only because

of our belief about the chances.}

\footnote{See for example \cite{Clark} who suggests that ergodic theory provides a way of interpreting probabilities in statistical mechanics.
  Actually very few systems have been proved to be ergodic although a great deal of effort has been devoted to attempting such proofs. See \cite{Sklar (1994)} for discussion.}

\footnote{Equivariance means that the dynamical laws maintain the preserve the probability distribution.
  I argue for this claim at length in ``David Lewis' Humean theory of Objective Chance.''}

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