

## Normalization

Let  $q$  and  $q_0$  represent a dimensional quantity and its reference value, respectively; its normalization is given by  $\tilde{q} = q/q_0$ . We select the reference values for the stress and strain to be  $\sigma_0 = 10^8$  ( $N/m^2$ ) and  $\epsilon_0 = 10^{-3}$ , respectively. The reference values of other quantities are determined in terms of these two reference variables such that all the normalized governing equations remain exactly the same form as the original equations listed in Section 2. For example, the reference values of the compliance and displacement are given by  $s_0 = \epsilon_0/\sigma_0 = 10^{-11}$  ( $m^2/N$ ) and  $u_0 = x_0\epsilon_0 = 10^{-3}x_0$ , where  $x_0$  is the characteristic length of the problem. Further, the reference value of the stress intensity factor is given by  $K_0 = \sigma_0\sqrt{x_0} = 10^8\sqrt{x_0}$  ( $N/m^{3/2}$ ). In summary, in the non-dimensional coordinate system given by

$$\tilde{x} = \frac{x}{x_0}, \quad \tilde{y} = \frac{y}{x_0}, \quad (1)$$

we consider materials with non-dimensional compliance coefficients

$$\tilde{s} = \frac{s \text{ (N/m}^2\text{)}}{10^{-11} \text{ (N/m}^2\text{)}}. \quad (2)$$

The non-dimensional displacement, stress and stress intensity factors are given by,

$$\tilde{u} = \frac{u \text{ (m)}}{10^{-3}x_0 \text{ (m)}}, \quad \tilde{\sigma} = \frac{\sigma \text{ (N/m}^2\text{)}}{10^8 \text{ (N/m}^2\text{)}}, \quad \tilde{K}_I + i\tilde{K}_{II} = \frac{K_I + iK_{II} \text{ (N/m}^{3/2}\text{)}}{10^8\sqrt{x_0} \text{ (N/m}^{3/2}\text{)}}. \quad (3)$$

All equations in Section 2 are interpreted to be the normalized (or non-dimensional) equations if all the quantities are replaced by their normalized quantities. Note that the program reads the input data in the non-dimensional form so that the numerical calculation is performed with these non-dimensional quantities. The final results, however, are given in the dimensional form using the characteristic length  $x_0$ , provided as a part of the input, as shown in (3).