Examples

Material considered is aluminum crystal (cubic) with the compliance matrix given by

$$\mathbf{s} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ & & s_{33} & s_{34} & s_{35} & s_{36} \\ & & & s_{44} & s_{45} & s_{46} \\ & & & & s_{55} & s_{56} \\ & & & & & s_{66} \end{bmatrix} = \begin{bmatrix} 1.59 & -0.58 & -0.58 & 0 & 0 & 0 \\ & 1.59 & -0.58 & 0 & 0 & 0 \\ & & 1.59 & 0 & 0 & 0 \\ & & & 3.52 & 0 & 0 \\ & & & & 3.52 & 0 \\ & & & & & 3.52 \end{bmatrix}, \qquad (1)$$

in units of $10^{-11} m^2/N$ showing only the upper half of the symmetric matrix.

Stress intensity factors for three aligned parallel cracks (Fig. 1 with $2a/d = 0.8$) subject to the uniaxial tension in an infinite body are given in Table 1. The size of the crack tip element is $c = 1/16a$, where a is the half crack length of the cracks. A large square domain was used to simulate the infinite domain. The SIFs for double edge cracks in a square plate, as shown in Fig. 2(a), are given in Table 2. Exploiting the symmetry of the problem only one half of the plate was analyzed as shown in Figure 2(b) which shows the boundary element mesh (with 30 elements) used for the non crack boundary. For the crack tip singular element, the size of the crack tip element is $c = a/32$ with the asymmetric arrangement of the crack elements shown in Fig. 9 of MANUAL page. Third example is a kinked crack in a semi-infinite plate under uniaxial tension as shown in Fig. 3. The non-crack boundary of the semi-infinite body is modeled by 34 nonhomogeneous elements consisting of a gradually refined mesh (Fig. 4(a)) as we approach the crack mouth. Twelve crack elements (6 for each of the straight segments AB and BC) are introduced along the kinked crack, as shown in Fig. 4(b); the size of the singular crack tip element AA_1 is 1/8 of the near crack tip segment AB. The numerical results for the stress intensity factors obtained by the e-Handbook are given in Table 3 in the non-dimensional form. The input files used for the calculation are available as template files at the e-Handbook web site (http://rci.rutgers.edu/~denda/e-Handbook).

Detailed discussion on the accuracy of the SIF results has been given by Denda and Marante [1] according to which the typical relative error is on the order of 0.5 %.

References

[1] M. Denda and M.E. Marante. Mixed mode bem analysis of multiple curvilinear cracks in the general anisotropic solids by the crack tip singular element. *Int. J. Solids and Struct.*, Vol. 41(5-6):pp. 1473–1489, 2004.

Table 1: Stress intensity factors for aligned three parallel cracks (Fig. 1 with $2a/d = 0.8$) in an infinite body under tension σ. The values in the parentheses are taken from Denda [2].

	$\frac{K_I}{\sigma\sqrt{\pi a}}$	$\frac{K_{II}}{\sigma\sqrt{\pi a}}$	$\frac{K_{III}}{\sigma\sqrt{\pi a}}$
A	0.86048(0.86095)	0.04326(0.04314)	0.00000(0.00000)
B	0.86048(0.86095)	-0.04326(-0.04314)	0.00000(0.00000)
C	0.76784(0.76879)	0.00000(0.00000)	0.00000(0.00000)
D	0.76784(0.76879)	0.00000(0.00000)	0.00000(0.00000)
E	0.86048(0.86095)	-0.04326(-0.04314)	0.00000(0.00000)
F	0.86048(0.86095)	0.04326(0.04314)	0.00000(0.00000)

Table 2: Stress intensity factors for double edge cracks in a square plate ($H/W = 1$) under tension (Fig. 2), where $\eta(a/W, H/W) = K_I/\{\sigma\sqrt{2W\tan(\pi a/2W)}\}$.

a/W	$\eta(a/W, H/W)$
0.1	1.14
0.2	1.17
0.4	1.195
0.8	1.05

Table 3: Stress intensity factors for a kinked edge crack (Fig. 3 with $\theta_1 = 90°$,$\theta_2 = 45°$ and $c_2 = 1.0$) in a half-plane under tension.

c_1	$K_I/\sigma\sqrt{\pi b}$	$K_{II}/\sigma\sqrt{\pi b}$
0.25	0.71701	-0.36306
0.50	0.71706	-0.36277
0.75	0.71578	-0.363408
0.90	0.71680	-0.35540

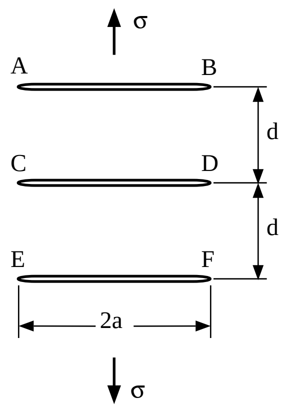

Figure 1: Three aligned parallel cracks under remote uniaxial tension.

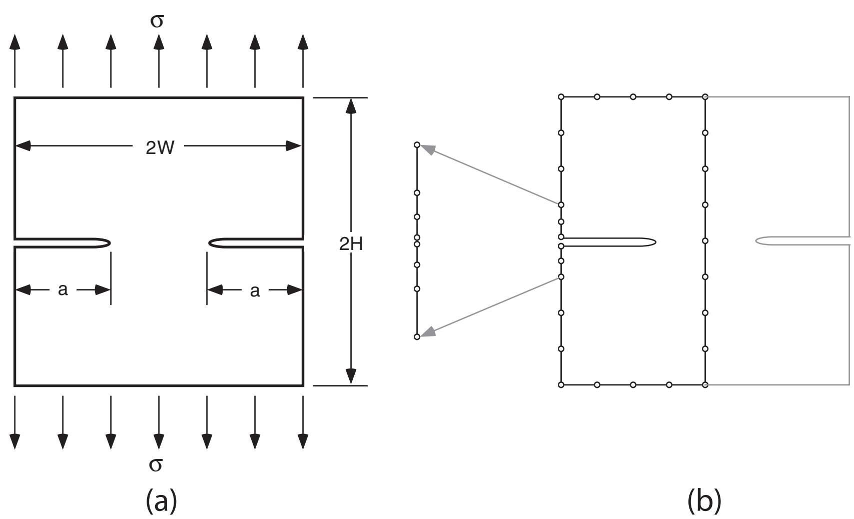

Figure 2: (a) Double edge cracks in a square plate ($W = H$) in tension. (b) Mesh used for double edge cracks in a square plate ($W = H$) in tension.

[2] M. Denda. Mixed mode I, II and III analysis of multiple cracks in plane anisotropic solids by the BEM: a dislocation and point force approach. *Int. J. of Engng. Anal. with Boundary Elements*, Vol. 25(4-5):pp. 267–278, 2001.

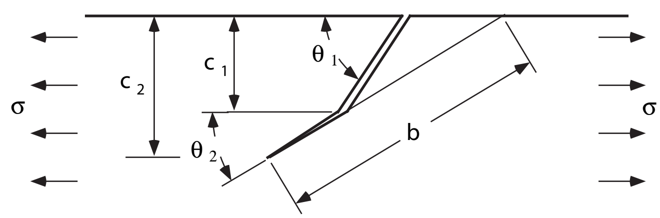

Figure 3: Kinked edge crack in a semi-infinite plate in tension.

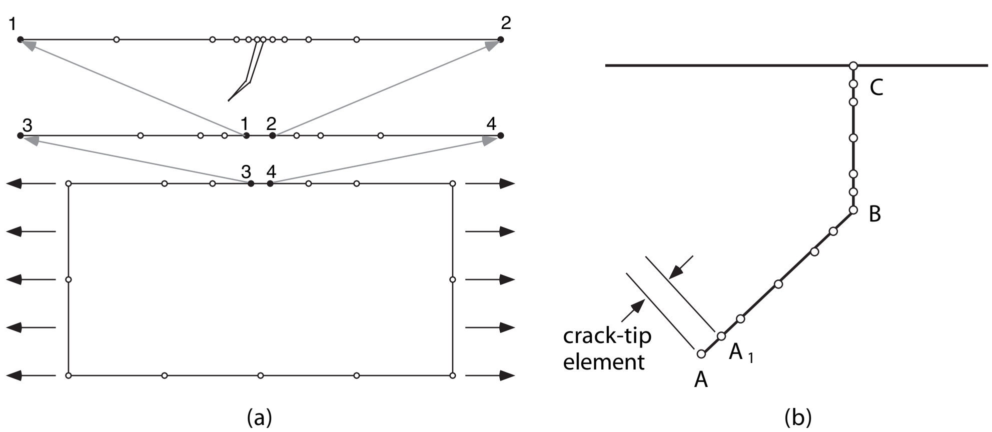

Figure 4: Mesh used for kinked edge crack in the semi-infinite body in tension: (a) Boundary mesh, (b) Crack mesh.