Quantifier-free least fixed point functions for phonology

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Introduction

- What kind of functions are phonological UR-SR maps?
- Automata-theoretic characterizations have focused on **subsequentiality** (Heinz and Lai, 2013; Payne, 2017; Chandlee and Heinz, 2018)
- **Logical** characterizations of *sets* provide representation-independent complexity hypotheses
- No previous logical characterizations of *functions* approach subsequentiality

- Chandlee and Lindell (forthcoming) capture **input strictly-local (ISL)** functions with **quantifier-free** (QF) logic
- We generalize this with **least fixed-point** extension of QF functions (QFLFP)
- QFLFP offers recursive, *output-based* definitions of functions
- This is a (proper?) subclass of the subsequential functions that tightly fits the typology of phonological functions

Logical definitions of functions

$\rtimes_1$	a_2	b_3	b_4	a_5	b_6	$\ltimes_7$

- **Model** of a string over Σ:
  - $D = \{1, 2, ..., n\}$ — $D = \{1, 2, 3, 4, 5, 6, 7\}$
  - $P_\sigma \subseteq D$ for each $\sigma \in \Sigma, \rtimes, \ltimes$ — $P_b = \{3, 4, 6\}$
  - A *predecessor* function p — $p(2) = 1$, etc.

$\rtimes_1$	a_2	b_3	b_4	a_5	b_6	$\ltimes_7$

- **QF logic** of strings:
  - **Terms** are
    - variables $x, y, ..., z$ range over D
    - $p(t)$ for term t
  - $P_\sigma(t)$ for $\sigma \in \Sigma, \rtimes, \ltimes$ and term t

$\rtimes_1$	a_2	b_3	b_4	a_5	b_6	$\ltimes_7$

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  - E.g., $P_b(x)$

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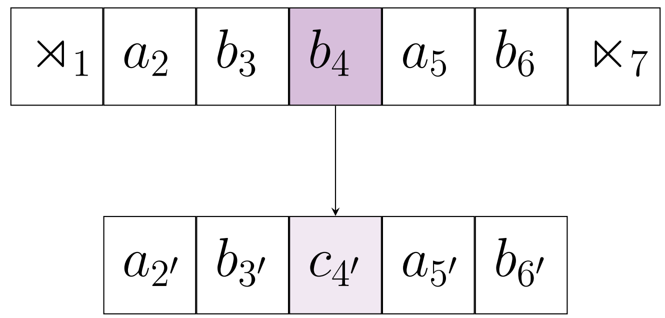
    - $p(t)$ for term t
  - $P_\sigma(t)$ for $\sigma \in \Sigma, \rtimes, \ltimes$ and term t
  - E.g., $P_b(x)$, $P_b(p(x))$

$\rtimes_1$	a_2	b_3	b_4	a_5	b_6	$\ltimes_7$

- **QF logic** of strings:
  - Syntax:

$$P_\sigma(t) \;\mid\; \neg\varphi \;\mid\; \varphi \vee \psi \;\mid\; \varphi \wedge \psi \;\mid\; \varphi \rightarrow \psi$$

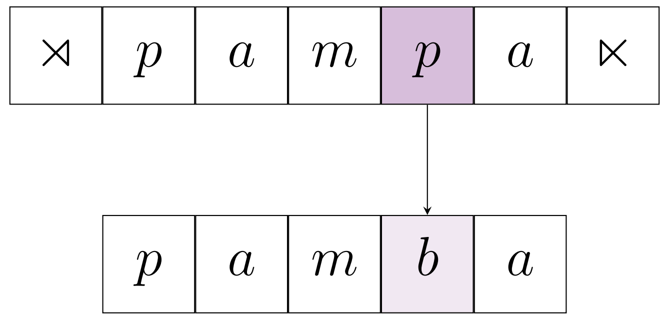
  - E.g., $P_b(x) \wedge P_b(p(x))$



- A **logical transduction** defines an output structure in the logic of the input structure (Courcelle, 1994; Courcelle et al., 2012)

$$\begin{aligned} P'_a(x) &\stackrel{\text{def}}{=} P_a(x) \\ P'_b(x) &\stackrel{\text{def}}{=} P_b(x) \wedge \neg(P_b(p(x))) \\ P'_c(x) &\stackrel{\text{def}}{=} P_b(x) \wedge (P_b(p(x))) \end{aligned}$$

- $b \to c$ / $b\_\_$

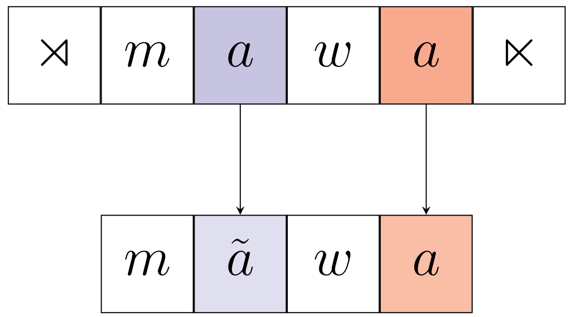


- Chandlee and Lindell (forthcoming): QF transductions capture ISL functions (Chandlee, 2014; Chandlee and Heinz, 2018)

$$
\begin{aligned}
P'_a(x) &\stackrel{\text{def}}{=} P_a(x) \\
P'_m(x) &\stackrel{\text{def}}{=} P_m(x) \\
P'_p(x) &\stackrel{\text{def}}{=} P_p(x) \land \neg(P_p(p(x))) \\
P'_b(x) &\stackrel{\text{def}}{=} P_p(x) \land (P_m(p(x)))
\end{aligned}
$$

- Long-distance patterns are not QF
- Iterative spreading, e.g. nasal spread in Malay (Onn, 1980)

/mawa/ $\rightarrow$ [mãw̃ã]

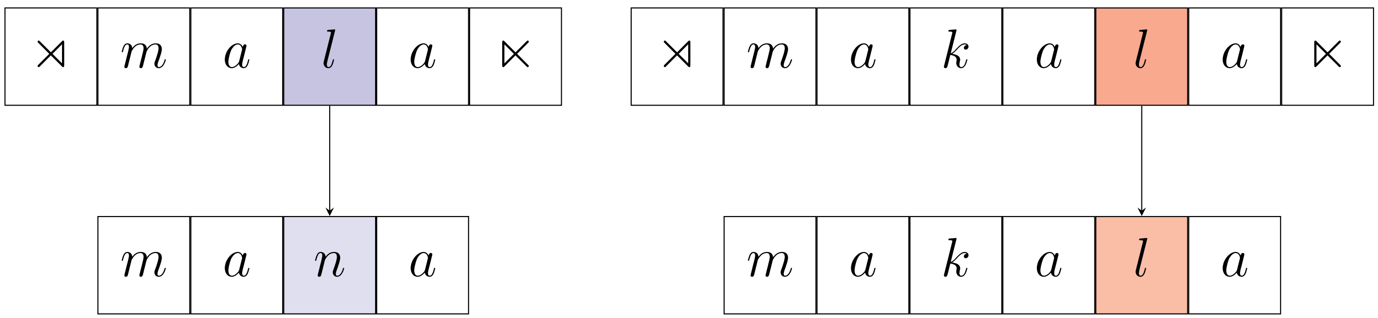


$$P'_{\tilde{a}}(x) \overset{\text{def}}{=} P_a(x) \wedge \texttt{nasal}(p(x))$$
$$P'_{a}(x) \overset{\text{def}}{=} P_a(x) \wedge \neg\texttt{nasal}(p(x))$$

- $\texttt{nasal}(x) \overset{\text{def}}{=} P_m(x) \vee P'_{\tilde{a}}(x) \vee P'_{\tilde{w}}(x)$

- Long-distance patterns are not QF
- L-D harmony, e.g. nasal harmony in Kikongo (Ao, 1991)

/mala/ $\rightarrow$ [mana]   /makala/ $\rightarrow$ [makana]



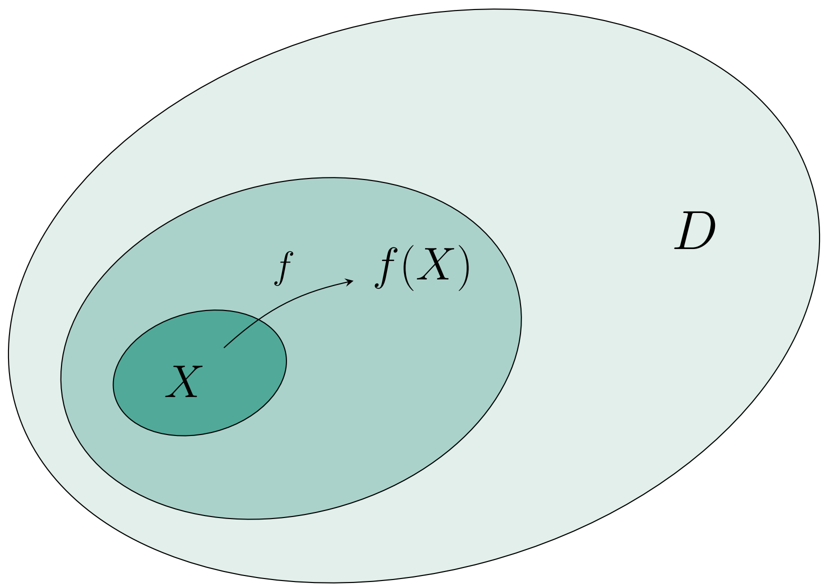
$$P'_n(x) \overset{\text{def}}{=} P_n(x) \vee \big(P_l \wedge \neg\mathtt{nasal}(p(p(x)))\big)$$
$$P'_l(x) \overset{\text{def}}{=} P_l(x) \wedge \neg\mathtt{nasal}(p(p(x)))$$

- $\mathtt{nasal}(x) \overset{\text{def}}{=} P_m(x) \vee P_n(x)$

- **Least-fixed point** logic allows:
  - reference to output structures;
  - definition of precedence from predecessor (p)
- Restriction to QF keeps logic weak

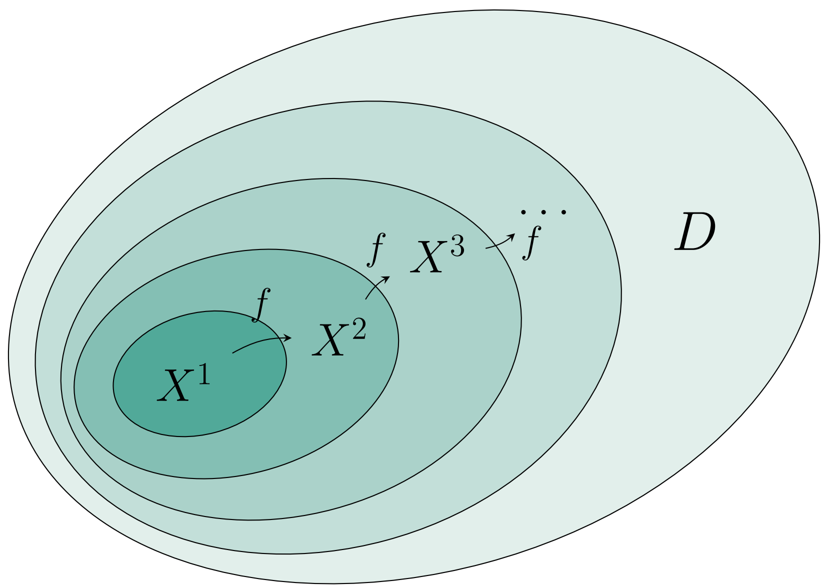
Least fixed point logic

- An **operator** on D is a function $f : \mathcal{P}(D) \to \mathcal{P}(D)$



- The **least fixed point** of f is $\texttt{lfp}(f) = \bigcup_i X^i$, where

$$X^0 = \emptyset, X^{i+1} = f(X^i)$$



- $\varphi(A, x)$ with a special predicate $A(x)$ induces an operator

$$f_\varphi(X) = \left\{ d \in D \;\middle|\; \varphi(A, x) \text{ is satisfied with } A \mapsto X, d \mapsto x \right\}$$

Example

$\rtimes_1$	a_2	b_3	a_4	a_5	a_6	c_7	a_8	$\ltimes_9$

$$\varphi(A, x) = P_a(x) \wedge \big(P_b(p(x)) \vee A(p(x))\big)$$

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$$\varphi(A, x) = P_a(x) \wedge \big(P_b(p(x)) \vee A(p(x))\big)$$

$$f_\varphi(\emptyset) \; = \; \{4\}$$

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$$\varphi(A, x) = P_a(x) \wedge \big(P_b(p(x)) \vee A(p(x))\big)$$

$$\begin{aligned}
f_\varphi(\emptyset) &= \{4\} && X^1 \\
f_\varphi(\{4\}) &= \{4, 5\} && X^2 \\
f_\varphi(\{4, 5\}) &= \{4, 5, 6\} && X^3 \\
f_\varphi(\{4, 5, 6\}) &= \{4, 5, 6\} && X^4 = X^5 = \ldots
\end{aligned}$$

$$\texttt{lfp}(f_\varphi) = \{4, 5, 6\}$$

- $\varphi(A, x)$ with a special predicate $A(x)$ induces an operator

$$f_\varphi(X) = \left\{ d \in D \;\middle|\; \varphi(A, x) \text{ is satisfied with } A \mapsto X, d \mapsto x \right\}$$

- QFLFP is QF extended with predicates of the form

$$\left[_{\texttt{lfp}} \varphi(A, x)\right](x)$$

  for some $\varphi(A, x)$ in QF extended with $A(x)$

$$\left[_{\texttt{lfp}} P_a(x) \wedge \big(P_b(p(x)) \vee A(p(x))\big)\right](x)$$

$\rtimes_1$	a_2	b_3	a_4	a_5	a_6	c_7	a_8	$\ltimes_9$

Example: iterative spreading with blocking

baaa $\mapsto$ bbbb
baaca $\mapsto$ bbbca
baacaba $\mapsto$ bbbcabb

$$P'_b(x) \overset{\text{def}}{=} [\mathtt{lfp}(P_b(x) \vee (A(p(x)) \wedge \neg P_c(x)))](x)$$

$\rtimes_1$	b_2	a_3	a_4	c_5	a_6	b_7	a_8	$\ltimes_9$

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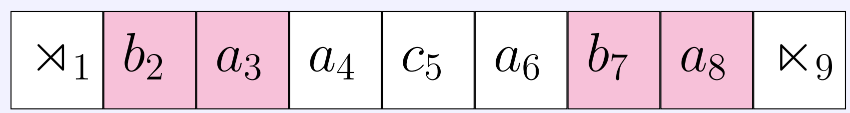

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$\rtimes_1$	b_2	a_3	a_4	c_5	a_6	b_7	a_8	$\ltimes_9$

b_2	b_3	b_4	c_5	a_6	b_7	b_8

Long-distance agreement

cbccca $\mapsto$ cbcccb

$$P'_b(x) \overset{\text{def}}{=} [\mathtt{lfp}(P_b(x) \vee A(p(x)))](x) \wedge \neg P_c(x)$$

$\rtimes_1$	c_2	b_3	c_4	c_5	c_6	a_7	$\ltimes_8$

c_2	b_3	c_4	c_5	c_6	b_7

Spreading with blocking:

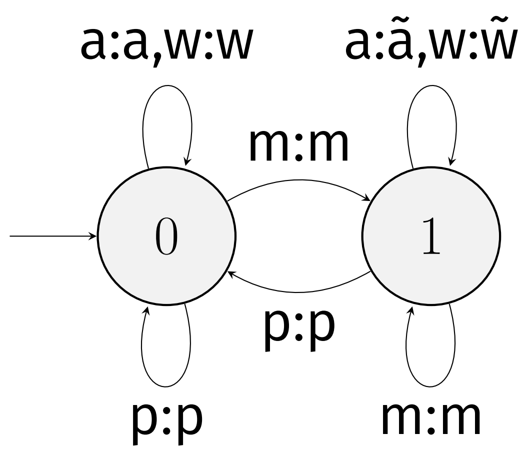
$$P'_b(x) \overset{\text{def}}{=} [_{\texttt{lfp}}(P_b(x) \vee (A(p(x)) \wedge \neg \boldsymbol{P_c(x)}))](x)$$

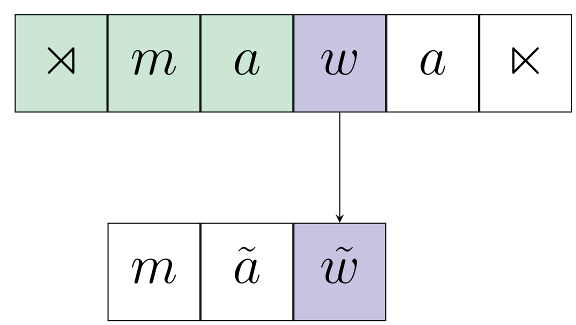
LD agreement:

$$P'_b(x) \overset{\text{def}}{=} [_{\texttt{lfp}}(P_b(x) \vee A(p(x)))](x) \wedge \neg \boldsymbol{P_c(x)}$$

QFLFP **is (probably) subsequential**

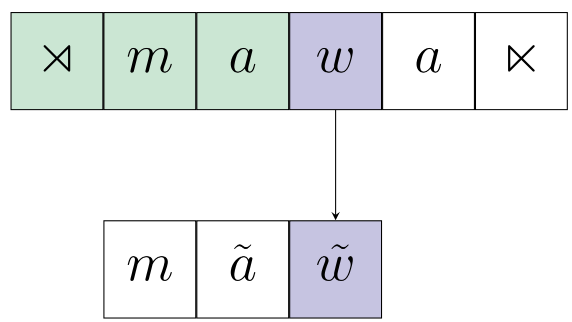
- **Subsequential functions** have some **deterministic** finite-state transducer (Schützenberger, 1977; Mohri, 1997)
- Reading left-to-right, we *immediately* know the output at each position in the input





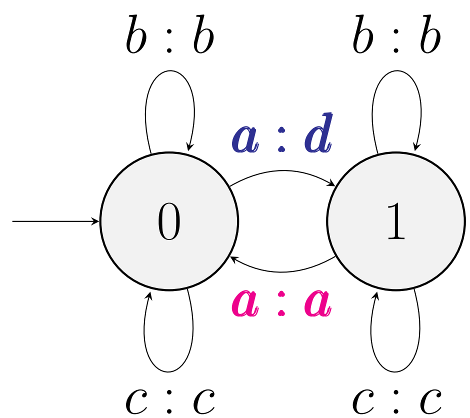
- For any $\varphi(x) \in \mathrm{QFLFP}$, whether a position satisfies $\varphi(x)$ depends entirely on the *preceding* information in the input
- Reading left-to-right, we *immediately* know the output at each position in the input

$$P_{\tilde{w}}(x) \stackrel{\text{def}}{=} \big[_{\texttt{lfp}} +\texttt{son}(x) \wedge \big(P_m(p(x)) \vee A(p(x))\big)\big](x)$$



Subsequential is (probably) not QFLFP

- Keeping track of even and odd-numbered elements of a particular type over arbitrary distances is subsequential



$bbbbbba \mapsto bbbbbbd$

$abbbbba \mapsto dbbbbba$

- We cannot think of a QFLFP definition for this function

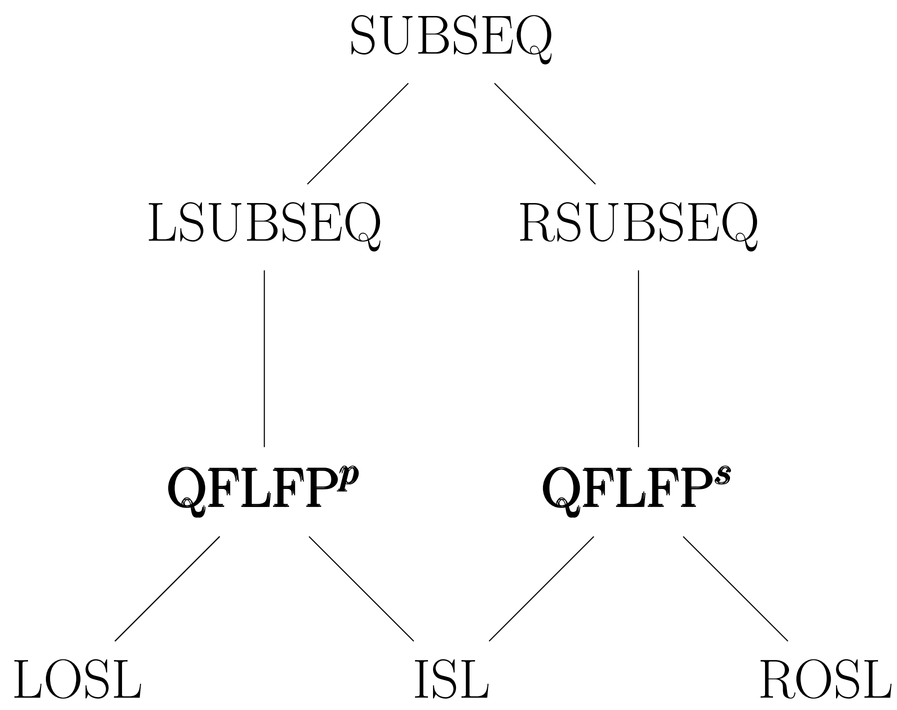
- This is a good *phonological* prediction of QFLFP; functions like “odd-numbered sibilants harmonize” are not attested.

- *But*, QFLFP *can* capture ‘local’ even/odd counting (for, e.g., iterative stress)

$$\left[_{\texttt{lfp}} \rtimes (p(x)) \vee A(p(p(x))\right](x)$$

$\rtimes_1$	a_2	a_3	a_4	a_5	a_6	a_7	a_8	$\ltimes_9$

The general picture (probably)



OSL = output strictly local functions (Chandlee, 2014; Chandlee et al., 2015)

Discussion

- QFLFP is a restrictive theory for phonology based on recursive definitions of local structures
- If $\mathrm{QFLFP} \subseteq \mathrm{SUBSEQ}$, then it is learnable (Oncina et al., 1993)
- Abstract definition of QFLFP?
- More efficient/plausible learner for QFLFP?

- Logic can be applied to non-string structures:
  - Features
  - Autosegmental representations
  - Metrical structure
  - Others?
- What do we get with two-place predicates and QFLFP (Koser et al., AMP)?

Conclusion

- QFLFP combines the restrictiveness of QF with the ability to recursively reference the output structure.
- Allows us to model non-ISL phenomena such as LD agreement and iterative spreading.
- This class of functions appears to cross-cut several subregular classes that have been applied to the modeling of phonological processes.
- If/as a subset of subsequential, it is also learnable.

Acknowledgements

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